A comparison of static and dynamic portfolio policies

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Abstract

Garleanu and Pedersen (2013) show that the optimal static portfolio policy in light of quadratic transaction costs is a weighted average of the existing portfolio and the target portfolio. In this paper, we demonstrate the importance of the robust target portfolio in the static portfolio policy that considers quadratic transaction costs. By using both empirical and simulated data, we find no evidence that the optimal dynamic portfolio policy proposed by Garleanu and Pedersen (2013) is superior to the static portfolio policy that trades towards the robust target portfolio. The robust target portfolio is achieved by either introducing time-varying covariances or restricting portfolio weights. Furthermore, the static portfolio with time-varying covariances and the short sale-constrained static portfolio are both very efficient in reducing portfolio turnover. The good performance of the static portfolio policy is robust to parameter uncertainty and trading parameters.

Key Words: Dynamic/Static portfolio policy, time-varying covariances, transaction costs.

JEL Classification Codes: G11, G17.

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1 Introduction

An investor who needs to trade off investment and consumption at multiple future dates will prefer a dynamic portfolio policy. In this regard, a static investor is sub-optimal in comparison with an investor who is able to make his/her decisions based on multiple periods ahead (Fabozzi et al., 2010). The short-sighted behaviour compiled in Markowitz (1952) has recently spawned a series of studies that have investigated dynamic mean-variance portfolio policies (see, e.g., Garleanu and Pedersen, 2013; Collin-Dufresne et al., 2015; Moallemi and Saglam, 2015; DeMiguel et al., 2016).

For instance, Garleanu and Pedersen (2013) (GP) derive an optimal dynamic portfolio policy when an investor faces costly trading and predictable returns. The optimal policy is the weighted average of the existing portfolio, the current Markowitz portfolio (moving target), and the expected Markowitz portfolios at all future dates (future trajectories). Under their framework, an investor should always aim in front of the moving target and trade smoothly towards the new target. Empirically, they document that this optimal portfolio policy beats the static portfolio policy in the presence of quadratic transaction costs.

Based on an objective function similar to that used by GP, DeMiguel et al. (2016) (DMN) develop their own analytical solution to the multi-period mean-variance portfolio policy.[1] Specifically, their strategy is a linear combination of the Markowitz portfolio (target portfolio), the previous period portfolio, and the next period portfolio. Instead of assuming predictable price changes, DMN propose a simpler scenario wherein the price changes of each asset are i.i.d.. This is a common assumption used in most of the transaction costs literature (see, e.g., Davis and Norman, 1990; Hong and Loewenstein, 2002; Hong, 2004). Under this assumption, the target portfolio is a fixed target instead of a moving target. They demonstrate that the dynamic optimal portfolio policy outperforms the static portfolio policy even though the latter takes transaction costs into consideration.

In this paper, we identify several findings that emphasise the significance of the robust target portfolio in the static portfolio policy. This robust target portfolio in achieved by

[1] We are very grateful to Xiaoling Mei for giving detailed explanations of their DMN model.

either introducing the variation in the covariance matrix or restricting portfolio weights. These two techniques are documented to yield good empirical features in limiting portfolio turnovers and generating satisfactory portfolio performance (see, e.g., Fleming et al., 2001; Jagannathan and Ma, 2003; DeMiguel et al., 2009a; DeMiguel et al., 2009b). Importantly, past theoretical and empirical evidence shows that the estimate on the second moment demonstrates a higher level of accuracy than the estimate on the first moment (see, e.g., Merton, 1980; Jagannathan and Ma, 2003). Therefore, the portfolio generated using the forecast on the covariance matrix can reasonably be expected to entail neither excessive transaction costs nor poor risk-adjusted performance. In fact, we show that under all circumstances examined, the static portfolio policy with time-varying covariances is able to perform at least on par with the GP-type[2] dynamic portfolio policy in the presence of quadratic transaction costs. The static portfolio policy can also be very efficient in limiting portfolio turnover.

By using commodity futures, iShares ETFs, and simulated data, we compare two GP-type dynamic portfolio policies with the static portfolio policies that have either time-varying returns or time-varying covariances. We also impose reasonable restrictions on asset positions to observe whether the performance of the static portfolio improves.[3] Further, we conduct numerous robustness checks; for instance, we use different trading parameters and use shrinkage approaches on the covariance matrix.

This paper contributes to the existing literature in the following ways. First, we find evidence that the static portfolio policy that trades towards a target portfolio with time-varying covariances performs as well as the dynamic portfolio policy in terms of the Sharpe, omega, and reward-to-risk ratios. To the best of our knowledge, such findings have not been reported in previous studies. Specifically, we apply six techniques to forecast the covariance matrix, namely, RiskMetrics, RiskMetrics2006, CCC-GJR-GARCH, DCC-GJR-GARCH, RBEKK,

[2]A GP-type policy refers to the dynamic optimal portfolio policy proposed by GP and its modified form in DMN.

[3]Despite the attractive features on constrained asset positions, sophisticated investors in reality do employ strategies that place no constraints on asset positions, e.g., equity long-short strategy.

and RDCC-GJR-GARCH.[4] Interestingly, the static portfolio policy with time-varying covariances predicted by each of the techniques performs at least on par with the GP and DMN portfolio policies before and after transaction costs for all cases examined. Moreover, our static portfolios generate lower transaction costs than the GP dynamic portfolio policy. Unlike other horse race studies, we show that the predictability of the model inputs is irrelevant to the relative performance presented.[5] Indeed, this good performance with respect to both the net performance and trading costs holds even when the model inputs in the dynamic portfolio policy have a higher degree of forecastability. Additionally, we show that the trading parameters (i.e. the transaction cost parameter λ and risk aversion parameter γ) are not the main drivers of the performance differences.

Second, with predictable returns, we find no evidence that the dynamic portfolio policy is superior to the static portfolio policy with constrained trading positions. Specifically, we find that over a relatively long sample period, when returns in both portfolios are estimated by the same technique, such as a characteristic-based model, the norm-constrained[6] static portfolio policy generates competitive portfolio performance compared with the GP dynamic portfolio policy after transaction costs without incurring excessive turnover.

Third, we extend comparisons from commodity futures to country ETFs. Existing studies (see, e.g., GP; DeMiguel et al., 2015; DMN) present portfolio performance by mainly relying on commodity futures data. By contrast, we use iShares country ETF data issued for the G6[7] countries and calibrate their transaction cost parameter λ and risk aversion parameter γ. Although the data characteristics of country ETFs and their corresponding basic parameter

[4]CCC-GJR-GARCH stands for the constant conditional correlation Glosten-Jagannathan-Runkle (Glosten et al., 1993) generalised autoregressive conditional heteroscedasticity model. DCC-GJR-GARCH stands for the dynamic conditional correlation Glosten-Jagannathan-Runkle (Glosten et al., 1993) generalised autoregressive conditional heteroscedasticity model. RBEKK stands for the rotated Baba-Engle-Kraft-Kroner (Engle and Kroner, 1995) model. RDCC-GJR-GARCH stands for the rotated dynamic conditional correlation Glosten-Jagannathan-Runkle (Glosten et al., 1993) generalised autoregressive conditional heteroscedasticity model.

[5]To be consistent with the portfolio selection literature, we conduct a horse race study and employ portfolio strategies and datasets similar to those used by GP and DMN.

[6]By 'norm constrained' is meant that the norm of the portfolio-weight vector is lower than a threshold. In this paper, the absolute value of each asset weight is lower than a numerical value. The short-sale constraint can be treated as a special form of norm constraint.

[7]There was no corresponding US ETF index when the World Equity Benchmark Series (WEBS) were first launched in 1996.

settings are substantially different from those of commodity futures, we observe patterns with respect to ETFs that consistently support our conclusions.

The rest of the paper is organised as follows. Section 2 demonstrates the theoretical frameworks for several portfolio policies and evaluation algorithms. Section 3 provides the data used in the analysis. Section 4 shows the empirical and simulation results, the graphs of the time-series asset positions, and the robustness checks. Section 5 concludes.

2 Conservative trading strategies

Assume that institutional investors can be divided into static (one-period) institutional investors who maximise the single-period mean-variance utility and long-term institutional investors who maximise the multi-period mean-variance utility. They are assumed to focus on the utility of wealth changes instead of consumption, and they employ *price changes* in excess of the risk-free rate instead of *excess returns*.[8] The excess price changes between t and $t+1$ are computed as $\boldsymbol{p}_{t+1} - (1 + r_f)\,\boldsymbol{p}_t$. Similar to GP and DeMiguel et al. (2015), we consider a relatively large scale of funds under management. With such a large amount circulating in the market, a distortion can be reasonably expected in market prices. To measure the influence on traded asset prices, we follow GP: the market price impact is linear in terms of the number of contracts or shares traded. The transaction costs associated with trading $\Delta\boldsymbol{x}_t$ contracts or shares are thus constructed as $\frac{1}{2}\Delta\boldsymbol{x}_t'\boldsymbol{\Lambda}\Delta\boldsymbol{x}_t$, where $\boldsymbol{\Lambda}_{N\times N}$ is the symmetric positive-definite matrix (a multi-dimensional version of Kyle's lambda (Kyle, 1985)), reflecting the sensitivities of transaction costs to the number of contracts or shares traded. In addition, we allow the cost of executing trades to be proportional to the covariance of price changes, as shown by Greenwood (2005) and Engle and Ferstenberg (2007). We simplify the expression by letting $\boldsymbol{\Lambda} = \lambda\boldsymbol{\Sigma}$, where λ is set to a positive scalar.

[8] Using price changes instead of returns avoids tracking the risky-asset price evolution and makes it easier to derive a closed-form solution. However, unscaled price changes may lead to extremely volatile time series compared with returns.

2.1 Dynamic portfolio policies

Two dynamic portfolio policies have been shown to have obvious advantages over the static portfolio policy: (1) the dynamic trading strategy developed by GP; (2) its modified version derived by DMN.

2.1.1 GP dynamic optimisation

GP develop an optimal dynamic portfolio policy under the assumption that trading is costly and that asset returns can be estimated by rolling Sharpe ratios over different horizons. Different from the traditional one-period mean-variance objective function, the augmented objective function maximises the present value of expected future mean-variance utilities. To ensure that the analysis is more in line with real-world trading, they define the mean-variance utilities as gains in price changes penalised for risks and quadratic transaction costs. The dynamic trading strategies are designed as follows:

$$\max_{\boldsymbol{x}_0,\boldsymbol{x}_1,\dots} E_0\left[\sum_{t=0}^{\infty}(1-\rho)^{t+1}\left(\boldsymbol{x}_t'\boldsymbol{r}_{t+1}-\frac{\gamma}{2}\boldsymbol{x}_t'\boldsymbol{\Sigma}\boldsymbol{x}_t\right)-\frac{(1-\rho)^t}{2}\Delta\boldsymbol{x}_t'\boldsymbol{\Lambda}\Delta\boldsymbol{x}_t\right], \tag{1}$$

where $0<\rho<1$ is an investor's impatience factor, γ is the absolute risk aversion parameter, and $\boldsymbol{x}_t$ represents a vector of the number of contracts or shares traded.

Because future price information can be detected from the characteristics of assets, a characteristic-based model is constructed to forecast the expected price changes. The characteristics of each asset are represented by the rolling Sharpe ratios over three different horizons: the past five days, the past one year, and the past five years. To avoid dividing a standard deviation that is close to zero, the standard deviations are winsorised below the 10th percentile. Further, the estimated excess price change of security s is given by

$$r_{t+1}^s=\beta^0+\beta^1 f_t^{5d,s}+\beta^2 f_t^{1y,s}+\beta^3 f_t^{5y,s}+\epsilon_{t+1}^s, \tag{2}$$

where $f_t^{5d,s}$ is the past five day Sharpe ratio, $f_t^{1y,s}$ is the past one year Sharpe ratio, and $f_t^{5y,s}$ is the past five year Sharpe ratio.

Another innovation applied in constructing the price change forecast is the introduction of the mean-reversion speed (alpha decay). GP assume that the dynamic structure of $f_t^{i,s}$ depends on only its own past levels. Put differently, the characteristic i has the same alpha decay for every security in the portfolio; that is, for all s,

$$\Delta f_{t+1}^{i,s} = -\varpi^i f_t^{i,s} + \varrho_{t+1}^{i,s}, \tag{3}$$

where ϖ^i is the mean-reversion rate of the factor.

GP show that predictors with faster alpha decay have less weight in the target portfolio and vice versa. They believe that these alpha decays are important for enabling the dominance of dynamic optimisation over the best possible static optimisation. They further empirically show that the dynamic strategy consistently beats the static strategy regardless of which combination of γ and λ appears for the Markowitz portfolio in the static optimisation.

Implementing dynamic programming to solve Equation 1, GP derive the optimal portfolio as the weighted average of the current portfolio and the target portfolio ($\boldsymbol{x}_t^{AIM}$),

$$\boldsymbol{x}_t = \left(1 - \varphi^{DT}\right) \boldsymbol{x}_{t-1} + \varphi^{DT} \boldsymbol{x}_t^{AIM}, \tag{4}$$

where φ^{DT} is the optimal trading rate and equals $\frac{\alpha}{\lambda}$. Here, $\alpha = \frac{-[\gamma(1-\rho)+\lambda\rho]+\sqrt{[\gamma(1-\rho)+\lambda\rho]^2+4\gamma\lambda(1-\rho)^2}}{2(1-\rho)}$.

The target portfolio is defined as the weighted average of the current Markowitz portfolio ($\boldsymbol{x}_t^{MKTZ}$) and the expected Markowitz portfolios at all future dates,

$$\boldsymbol{x}_t^{AIM} = \sum_{i=t}^{\infty} z\,(1-z)^{i-t} E_t\left(\boldsymbol{x}_i^{MKTZ}\right). \tag{5}$$

With returns estimated by the characteristic-based model, GP show that the target portfolio resembles the Markowitz portfolio with factors scaled down on the basis of transaction costs, risk aversion, and the mean-reversion speed of the factors. Inspired by this observation, they derive a simple closed-form solution corresponding to the dynamic optimisation strategy,

$$\boldsymbol{x}_t = \left(1 - \varphi^{DT}\right) \boldsymbol{x}_{t-1} + \varphi^{DT} \left(\gamma \boldsymbol{\Sigma}\right)^{-1} \sum_{i=1}^{3} \frac{1}{1 + \varpi^i \alpha / \gamma} \beta^i f_t^i. \tag{6}$$

2.1.2 DMN portfolio policy

DMN consider an objective function that is very similar to that of GP. Instead of following an infinite horizon, they develop a portfolio trading strategy that assumes a finite investment horizon and that is consistent with GP's trajectory of the optimal portfolio. The trajectory of the optimal portfolio in DMN is shaped by the target, past, and future portfolios. The closed-form solution of the multi-period portfolio policy by DMN is given as

$$\boldsymbol{x}_t = \omega_1 \boldsymbol{x}^* + \omega_2 \boldsymbol{x}_{t-1} + \omega_3 \boldsymbol{x}_{t+1} \quad \forall t \in \{1, 2, \ldots, T-1\}, \tag{7}$$

where $\omega_1 + \omega_2 + \omega_3 = 1$ with $\omega_1 = \left[(1-\rho)\gamma\right] / \left[(1-\rho)\gamma + \lambda + (1-\rho)\lambda\right], \omega_2 = \lambda / \left[(1-\rho)\gamma + \lambda + (1-\rho)\lambda\right]$, and $\omega_3 = \left[(1-\rho)\lambda\right] / \left[(1-\rho)\gamma + \lambda + (1-\rho)\lambda\right]$, and $\boldsymbol{x}^*$ is the fixed target portfolio.

The next period portfolio, $\boldsymbol{x}_{t+1}$, is computed in line with the optimal portfolio solution given in GP. Recall that according to GP, the optimal portfolio is the weighted average of the existing and target portfolios:

$$\boldsymbol{x}_t = \left(1 - \frac{\alpha}{\lambda}\right) \boldsymbol{x}_{t-1} + \frac{\alpha}{\lambda} \left[\sum_{i=t}^{\infty} z \left(1 - z\right)^{i-t} E_t \left(\boldsymbol{x}_i^{MKTZ}\right)\right]. \tag{8}$$

By assuming that investments after the finite investment horizon are purely in the category of risk-free assets such as bonds, DMN replace the positive infinity ∞ in Equation 8 with a finite length investment horizon T. Therefore, the optimal portfolio can be directly solved by using Equation 8. With the optimal portfolio in each period known, $\boldsymbol{x}_{t+1}$ is known, and it can be inserted back into Equation 7.

Upon reaching the last observation in the investment horizon, the optimal portfolio for the next period no longer exists. Therefore, the observation corresponding to time T is as follows:

$$\boldsymbol{x}_T = \theta_1 \boldsymbol{x}^* + \theta_2 \boldsymbol{x}_{T-1}, \tag{9}$$

where $\theta_1 + \theta_2 = 1$ with $\theta_1 = \left[(1-\rho)\gamma\right] / \left[(1-\rho)\gamma + \lambda\right]$ and $\theta_2 = \lambda / \left[(1-\rho)\gamma + \lambda\right]$.

2.2 Static portfolio policies

Two static portfolio policies are considered. The first model ('Markowitz') trades in each period, optimising the one-period mean-variance utility that ignores transaction costs. The second model ('Static optimisation') is a one-period mean-variance strategy that is subject to quadratic transaction costs.

The classical Markowitz portfolio considers an investor who ignores transaction costs ($\boldsymbol{\Lambda} = \boldsymbol{0}$) and exists for only one period. Given the mean-variance preference of Markowitz (1952), the optimal portfolio policy is as follows:

$$\boldsymbol{x}_t = (\gamma \boldsymbol{\Sigma})^{-1} E_t(\boldsymbol{r}_{t+1}). \tag{10}$$

GP modify the mean-variance utility in Markowitz (1952) by introducing a term that measures the transaction costs associated with large trades ($\boldsymbol{\Lambda} = \lambda \boldsymbol{\Sigma}$). Unlike the dynamic strategy (Equation 1), which aims to maximise the present value of future utilities, the static strategy maximises the excess price change net of risks and transaction costs in each period; that is,

$$\max_{\boldsymbol{x}_t} \boldsymbol{x}_t' E_t(\boldsymbol{r}_{t+1}) - \frac{\gamma}{2} \boldsymbol{x}_t' \boldsymbol{\Sigma} \boldsymbol{x}_t - \frac{\lambda}{2} \Delta \boldsymbol{x}_t' \boldsymbol{\Sigma} \Delta \boldsymbol{x}_t. \tag{11}$$

Differentiating it in terms of $\boldsymbol{x}_t$, we have the closed-form solution for the static portfolio policy subject to transaction costs,

$$\boldsymbol{x}_t = \left(1 - \varphi^{ST}\right) \boldsymbol{x}_{t-1} + \varphi^{ST} \left(\gamma \boldsymbol{\Sigma}\right)^{-1} E_t \left(\boldsymbol{r}_{t+1}\right), \tag{12}$$

where $\varphi^{ST} = \frac{\gamma}{\gamma+\lambda}$.

In contrast to the settings adopted in GP, we vary the covariance matrix over time while fixing the expected price change. In each period, the expected price change is fixed as the mean price change across the entire sample period. By doing so, the constant expected price change limits the excessive turnover. Reasonable predictability is also guaranteed because of the nature of the in-sample forecast conducted in this paper. Therefore, we set $E_t \left(\boldsymbol{r}_{t+1}\right) = \boldsymbol{\mu}$ in Equations 10, 11, and 12. In addition, as the covariance matrix becomes time varying, we replace $\boldsymbol{\Sigma}$ with $\boldsymbol{\Sigma}_t$ in Equations 10, 11, and 12.

The in-sample analysis enables us to focus on the economic insights underlying the portfolio construction without worrying too much about the predictability of each technique employed. The predictability discrepancy between the return and risk predictions can be reasonably expected to not be a major factor that drives the differences in portfolio performance.

2.3 Time-varying risk

We implement six commonly applied risk-forecasting techniques. They broadly cover three major types of risk estimation approaches. A brief introduction of each model is provided below.

2.3.1 EWMA-type models

Compared with a moving average covariance estimator, the *exponentially weighted moving average (EWMA or RiskMetrics)* approach allows more weight to be added to recent observations. The RiskMetrics approach uses the weighted average of the squared residuals and variance-covariance matrix in the previous period,

$$\mathbf{\Sigma}_t = (1-\varsigma)\,\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}'_{t-1} + \varsigma\mathbf{\Sigma}_{t-1}, \tag{13}$$

where $\boldsymbol{\epsilon}_{t-1}$ is a vector of the innovation terms from the mean equation in the previous period.

In contrast to the RiskMetrics approach, where weights on past returns exponentially decay, in the *RiskMetrics2006* approach, the weights hyperbolically decay. This methodology extends the RiskMetrics approach through the calibration of the volatility as a weighted sum of EWMAs:

$$\mathbf{\Sigma}_t = \sum_{i=1}^{n} \varrho_i \left[(1-\varsigma_i)\,\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}'_{t-1} + \varsigma_i\mathbf{\Sigma}_{i,t-1}\right], \tag{14}$$

where $\varrho_i = \frac{1}{C}\left[1 - \frac{\ln\left(\tau_i^{LDF}\right)}{\ln\left(\tau_0^{LDF}\right)}\right]$ with C being a normalisation constant so that $\sum_{i=1}^{n} \varrho_i = 1$. $\varsigma_i = \exp\left(-\frac{1}{\tau_i^{LDF}}\right)$, and $\tau_i^{LDF} = \tau_1^{LDF}\jmath^{i-1}$ for $i = 1, 2, \ldots, n$[9].

To initiate a RiskMetrics2006 process, we need initial values for the logarithmic decay factor τ_0^{LDF}, lower cut-off point τ_1^{LDF}, upper cut-off point $\tau_{\max}^{LDF}$, $\jmath$, and n.

2.3.2 Multivariate GARCH-type models

The *constant conditional correlation (CCC)* model, introduced by Bollerslev (1990), decomposes the conditional covariance into time-varying conditional variances and the time-invariant conditional correlation. Formally, it is as follows:

$$\mathbf{\Sigma}_t = \boldsymbol{D}_t\boldsymbol{R}\boldsymbol{D}_t, \tag{15}$$

where $\boldsymbol{D}_t = diag\left(\sqrt{h_{1,t}}, \ldots, \sqrt{h_{N,t}}\right)$ and $\boldsymbol{R}$ $(N \times N)$ is a correlation matrix. We measure the conditional variance of each security's return series by the *Glosten-Jagannathan-Runkle generalised autoregressive conditional heteroscedasticity (GJR-GARCH)* model proposed by Glosten et al. (1993), which accounts for the asymmetric effects of return shocks. A GJR-GARCH (I,J,K) takes the following form:

[9] In the RiskMetrics2006, $n = \dfrac{\ln\left(\tau_{max}^{LDF}/\tau_1^{LDF}\right)}{\ln \jmath}$.

$$h_{p,t} = \omega_i + \sum_{i=1}^{I} a_i \epsilon_{t-i}^2 + \sum_{j=1}^{J} \phi_j \epsilon_{t-j}^2 I_{[\epsilon_{t-j}<0]} + \sum_{k=1}^{K} b_k h_{t-k}, \tag{16}$$

where $I_{[\epsilon_{t-j}<0]} = \begin{cases} 1 & \epsilon_{t-j} < 0 \\ 0 & otherwise \end{cases}$.

The *dynamic conditional correlation (DCC)* model by Engle (2002) improves the CCC model by applying scalar Baba-Engle-Kraft-Kroner (BEKK)-like dynamics proposed by Engle and Kroner (1995) to the conditional correlations. After the GJR-GARCH model is fitted to each return series, the standardised residual $\vartheta_{p,t} = \epsilon_{p,t}/\sqrt{h_{p,t}}$ is incorporated into the following operations:

$$\boldsymbol{\Sigma}_t = \boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t, \tag{17}$$

$$\boldsymbol{R}_t = \boldsymbol{Q}_t^\star \boldsymbol{Q}_t \boldsymbol{Q}_t^\star, \tag{18}$$

$$\boldsymbol{Q}_t = (1 - a - b)\,\overline{\boldsymbol{Q}} + a \boldsymbol{\vartheta}_{t-1} \boldsymbol{\vartheta}_{t-1}' + b \boldsymbol{Q}_{t-1}, \tag{19}$$

where $\boldsymbol{Q}_t^\star = diag\left(\sqrt{q_{1,t}}, \ldots, \sqrt{q_{N,t}}\right)$, $q_{p,t}$ is the diagonal elements of $\boldsymbol{Q}_t$, $\overline{\boldsymbol{Q}}$ is the average correlation, and $\boldsymbol{\vartheta}_t$ is an $N \times 1$ vector of $\vartheta_{p,t}$.

2.3.3 Rotated ARCH models

Noureldin et al. (2014) propose two new classes of rotated-type multivariate volatility models. They extend the idea of variance targeting to covariance targeting in multivariate models of any dimension by rotating the raw returns. Specifically, for the *rotated Baba-Engle-Kraft-Kroner (RBEKK)* model, they rotate returns and then fit them via a BEKK-type parameterisation of time-varying covariances. First, by applying the spectral decomposition in the second equality, they decompose the unconditional covariance of asset raw returns to

$$\overline{\boldsymbol{\Sigma}} = \boldsymbol{P}\boldsymbol{\Pi}\boldsymbol{P}', \tag{20}$$

where $\boldsymbol{P}$ is a matrix of eigenvectors, and $\boldsymbol{\Pi}$ is the eigenvalue matrix. They then define the rotated returns as

$$\boldsymbol{e}_t = \boldsymbol{P}\boldsymbol{\Pi}^{-\frac{1}{2}}\boldsymbol{P}'\boldsymbol{r}_t. \tag{21}$$

Considering a covariance-targeting BEKK-type parameterisation (Engle and Kroner, 1995) for the conditional covariance of the rotated returns $\boldsymbol{G}_t$, they obtain

$$\boldsymbol{G}_t = (\boldsymbol{I}_N - \boldsymbol{A}\boldsymbol{A}' - \boldsymbol{B}\boldsymbol{B}') + \boldsymbol{A}\boldsymbol{e}_{t-1}\boldsymbol{e}'_{t-1}\boldsymbol{A}' + \boldsymbol{B}\boldsymbol{G}_{t-1}\boldsymbol{B}', \tag{22}$$

$$\boldsymbol{G}_0 = \boldsymbol{I}_N, \tag{23}$$

where $\boldsymbol{I}_N$ is an $N \times N$ identity matrix, $\boldsymbol{A}$ and $\boldsymbol{B}$ are conformable parameter matrices, and $(\boldsymbol{I}_N - \boldsymbol{A}\boldsymbol{A}' - \boldsymbol{B}\boldsymbol{B}')$ is positive semidefinite.

The RBEKK model above implies a constrained version of the BEKK model for unrotated returns, which can be easily observed when the conditional covariance matrix of unrotated returns from above is derived as follows:

$$\boldsymbol{\Sigma}_t = \overline{\boldsymbol{H}}^{\frac{1}{2}}\boldsymbol{G}_t\overline{\boldsymbol{H}}^{\frac{1}{2}} = \overline{\boldsymbol{C}}\overline{\boldsymbol{C}}' + \overline{\boldsymbol{A}}\boldsymbol{r}_{t-1}\boldsymbol{r}'_{t-1}\overline{\boldsymbol{A}}' + \overline{\boldsymbol{B}}\boldsymbol{\Sigma}_{t-1}\overline{\boldsymbol{B}}', \tag{24}$$

where $\overline{\boldsymbol{A}} = \overline{\boldsymbol{H}}^{\frac{1}{2}}\boldsymbol{A}\overline{\boldsymbol{H}}^{-\frac{1}{2}}, \overline{\boldsymbol{B}} = \overline{\boldsymbol{H}}^{\frac{1}{2}}\boldsymbol{B}\overline{\boldsymbol{H}}^{-\frac{1}{2}}$, and $\overline{\boldsymbol{C}}\overline{\boldsymbol{C}}' = \overline{\boldsymbol{H}}^{\frac{1}{2}}(\boldsymbol{I}_N - \boldsymbol{A}\boldsymbol{A}' - \boldsymbol{B}\boldsymbol{B}')\overline{\boldsymbol{H}}^{\frac{1}{2}}$. Equation 24 is a constrained version of the BEKK model for unrotated returns as $\overline{\boldsymbol{A}}$ and $\overline{\boldsymbol{B}}$ are rotations of $\boldsymbol{A}$ and $\boldsymbol{B}$.

The *rotated dynamic conditional correlation (RDCC)* model has a different method of deriving $\boldsymbol{Q}_t$ from that in Equation 19, as it introduces more flexibility than the traditional DCC model. By applying a covariance-targeting BEKK-type parameterisation, Noureldin et al. (2014) model the dynamics of the conditional covariance ($\boldsymbol{Q}^{\sharp}_t$) of rotated *standardised* returns ($\tilde{\boldsymbol{e}}_t = \boldsymbol{P}_C \boldsymbol{\Pi}_C^{-\frac{1}{2}} \boldsymbol{P}'_C \left(\boldsymbol{D}_t^{-1} \boldsymbol{r}_t\right)$) as follows:

$$\boldsymbol{Q}_t^{\sharp} = (\boldsymbol{I}_N - \boldsymbol{A}\boldsymbol{A}' - \boldsymbol{B}\boldsymbol{B}') + \boldsymbol{A}\tilde{\boldsymbol{e}}_{t-1}\tilde{\boldsymbol{e}}'_{t-1}\boldsymbol{A}' + \boldsymbol{B}\boldsymbol{Q}_{t-1}^{\sharp}\boldsymbol{B}', \tag{25}$$

$$\boldsymbol{Q}_0^{\sharp} = \boldsymbol{I}_N. \tag{26}$$

$\boldsymbol{Q}_t$ is then given as follows:

$$\boldsymbol{Q}_t = \boldsymbol{P}_C \boldsymbol{\Pi}_C^{\frac{1}{2}} \boldsymbol{P}'_C \boldsymbol{Q}_t^{\sharp} \boldsymbol{P}_C \boldsymbol{\Pi}_C^{\frac{1}{2}} \boldsymbol{P}'_C. \tag{27}$$

By following the DCC model as described in Equations 17 and 18, the conditional variance of raw returns can be achieved.

2.4 Norm-constrained portfolio policies

By 'norm constrained' is indicated the imposition of restrictions on asset positions, a technique widely applied in portfolio selection (see, e.g., Frost and Savarino, 1988; Jagannathan and Ma, 2003; DeMiguel et al., 2009a). Theoretical proofs show that doing so is equivalent to a shrinkage approach on the model input. More importantly, shrinking either the first-order or the second-order moments of asset returns largely reduces estimation error, leading to a significant improvement in portfolio performance. A short-sale constraint is very efficient in limiting portfolio turnover and transaction costs. By deterring short selling, extreme movements in positions are reduced to the minimum. In this paper, we apply both short-sale and norm constraints to the asset positions of the Markowitz portfolio subject to quadratic transaction costs.

For a short sale-constrained portfolio, we set the lower bound to zero without placing a limitation on how many contracts or shares an investor buys for each security. The implication is that $x_{i,t} \geq 0$ is a constraint function to Equation 11 for a short sale-constrained portfolio. For a norm-constrained portfolio, apart from placing the lower bound of zero on each asset position, we set the upper bound as the sum of the average asset positions across the sample period. The average asset positions are obtained from averaging the time-series asset positions in the previous short sale-constrained optimisation process. Hence, the only difference between the two constrained portfolios is the extra condition on the norm-constrained portfolio that limits maximum investments.

2.5 Evaluation algorithms

We evaluate the performance of each portfolio strategy both before and after transaction costs. To accurately judge each strategy in the case of losses, we use the omega ratio, the reward-to-risk ratio (the mean-value-at-risk ratio) in addition to the Sharpe ratio. Supposing that $\hat{\boldsymbol{x}}_t$ is the estimated number of contracts or shares for investing at time $t+1$, the portfolio gains net of quadratic transaction costs are as follows:

$$R_{t+1}^{net} = \hat{\boldsymbol{x}}_t' \boldsymbol{r}_{t+1} - \frac{\lambda}{1-\rho} \Delta \hat{\boldsymbol{x}}_t' \boldsymbol{\Sigma} \Delta \hat{\boldsymbol{x}}_t \quad \forall t \in \{0, 1, 2, \ldots, T-1\}. \tag{28}$$

The net Sharpe ratio is then computed as the average of net portfolio gains across the entire sample divided by the standard deviation of the net price changes. Note that price changes are used to compute the Sharpe ratio instead of returns. The expression of the net Sharpe ratio is as follows:

$$SR^{net} = \frac{\bar{R}^{net}}{\sqrt{\frac{1}{T} \sum_{t=0}^{T-1} \left(R_{t+1}^{net} - \bar{R}^{net} \right)^2}}. \tag{29}$$

For the gross Sharpe ratio, we insert the price changes before considering transaction costs. The statistical test for the Sharpe ratio difference between each portfolio policy and the GP dynamic portfolio policy is the methodology suggested in Ledoit and Wolf (2008).

Introduced by Keating and Shadwick (2002), the omega ratio considers gains and losses separately without a previous assumption regarding a specific distribution for asset returns.

It is defined as the probability weighted ratio of gains to losses with reference to a threshold set by users. In this paper, we set the threshold to zero, which means zero price changes in excess of the risk-free rate. Similar to the Sharpe ratio, price changes are used to compute the omega ratio instead of returns. The numerical form of the omega ratio with respect to net price changes is as follows:

$$OR^{net} = \frac{E[R_{t+1}^{net} \mid R_{t+1}^{net} \geq 0]}{E[R_{t+1}^{net} \mid R_{t+1}^{net} \leq 0]}. \tag{30}$$

The gross omega ratio has the same expression as above, except the net price changes are replaced by price changes before transaction costs.

We apply the mean-value-at-risk ratio because the variance may not be an accurate measure of portfolio risk.[10]With the confidence level (c) set as 95%, the net mean-value-at-risk ratio is defined as follows:

$$RR^{net} = \frac{\bar{R}^{net}}{VaR_{net}^{c}}. \tag{31}$$

Similarly, the gross reward-to-risk ratio replaces the net price changes with the price changes before transaction costs.

3 Data

We employ two empirical datasets. First, we use a commodity futures dataset.[11] The majority of the 11 commodity futures used are similar to those used by GP. These commodity futures are shown in Table 1. We exclude futures contracts of agriculture and livestock because they have tight price limits that may cause noise when the return predictor is modelled. Specifically, we consider five energy futures (i.e. WTI Crude Oil, Heating Oil, Gasoline, and Natural Gas from the New York Mercantile Exchange (NYMEX), and Brent

[10]We also look at the mean-conditional value-at-risk ratio of each portfolio strategy in all experiments. Because the pattern generated by the mean-conditional value-at-risk ratio is very similar to that of the mean-value-at-risk ratio, we only keep the results related to the mean-value-at-risk ratio to conserve space.

[11]Commodity futures data are used in this paper to maintain consistency with the dataset used by GP.

Crude Oil from the Intercontinental Exchange (ICE)); three soft futures (i.e. Sugar, Coffee, and Cocoa from the Intercontinental Exchange (ICE)); two precious metal futures (i.e. Gold from the New York Commodities Exchange (COMEX) and Platinum from the New York Mercantile Exchange (NYMEX)); and one base metal futures (i.e. Copper from the New York Mercantile Exchange (NYMEX)).

Insert Table 1 here

The total time span of our data extends from 07/05/1993 to 02/09/2014, and it covers a five-year pre-sample period from 07/05/1993 to 28/08/1998 and a 16-year sample period from 01/09/1998 to 02/09/2014. Hence, it is longer and more updated than the data used by GP. In total, we have 1300 pre-sample daily observations and 3881 sample daily observations.

Second, we consider the countries in the G7 that appeared in the first launch of the World Equity Benchmark Series. Except for the US ETF, the country ETFs of the other six nations launched on 18/03/1996.[12] As shown in Table 1, these countries are Canada, France, Germany, Italy, Japan, and the UK. In total, we obtain 1300 pre-sample daily observations ranging from 18/03/1996 to 11/05/2001 and 3326 sample daily observations ranging from 14/05/2001 to 02/09/2014. In Table 1, Japan appears to demonstrate the lowest average asset price and the highest annualised return volatility. By contrast, France is associated with the highest average asset price and Canada is associated with the lowest annualised return volatility.

4 Empirical results

We use the same parameter settings regarding the key trading variables as those used by GP. In particular, we set the impatience factor to $\rho = 1 - \exp(-0.02/260)$, which corresponds to a 2% annualised rate in a year of 260 trading days. Regarding the scalar λ in the transaction

[12] Because short selling is allowed on both commodity futures and ETFs, it is reasonable to use the dynamic portfolio policies proposed by GP and DMN that allow long/short asset positions.

cost matrix, we calibrate it by following the empirical finding reported by Breen et al. (2002) and Engle et al. (2012). Breen et al. (2002) propose a measure of price impact, which quantifies the change in a company's stock price associated with its observed net trading volume. Their measure addresses important aspects of liquidity, such as bid-ask spread and quoted depth, that cannot be captured by existing measures. Furthermore, by considering orders for NASDAQ stocks, Engle et al. (2012) find that trading 1.59% of the daily volume in a stock influences the daily price by 0.10%.[13] Hence, for any security q, its transaction cost parameter follows the following relationship:

$$1.59\% \times volume_q \times {}^{\lambda_q}/_2 \times \sigma_q^2 = 0.10\% \times price_q. \tag{32}$$

We can now compute λ_q for each asset according to its contract information provided in Table 1. In line with GP, we calibrate λ as the median across estimates of λ_q for each asset. The median corresponding to 11 commodity futures and six ETFs is approximately 7×10^{-8} and 3×10^{-5}, respectively. The median for commodity futures is slightly different from the median value of 5×10^{-7} used by GP. This is because we use a more updated and longer time span and the commodity futures analysed in this paper are not exactly the same as those that appear in GP. To reach similar weights (φ^{DT} and φ^{ST}) on the Markowitz portfolio as those set by GP, we set the absolute risk aversion as $\gamma = 10^{-10}$. This scale of absolute risk aversion corresponds to the relative risk aversion of one for an amount of 10 billion US dollars under management. Similarly, we calibrate the absolute risk aversion of our ETF data over the entire sample period and obtain an amount of 10^{-8}. This amount corresponds to a relatively smaller amount of funds under management, which is 100 million US dollars with the relative risk aversion of one.

For the variance-covariance matrix used in all models, we shrink the correlation by 50% towards zero for robustness. This adjustment is applied to the transaction cost matrix and the predicted variance-covariance matrix. In addition to the robustness benefit noted by

[13]Although the findings in Breen et al. (2002) and Engle et al. (2012) concern the prices and trading volumes of stocks, we use the findings to calibrate the transaction cost parameter corresponding to commodity futures and ETFs, in line with GP and DMN.

GP, the shrunk variance-covariance matrix can smooth extreme position variations across periods. In terms of the risk forecast, the covariance matrix of the full sample is set as the initial value in Equation 13. In Equation 14, the initial value is assigned as follows: $\tau_0^{LDF} = 1560$, $\tau_1^{LDF} = 4$, $\tau_{max}^{LDF} = 512$, and $\jmath = \sqrt{2}$.[14]

4.1 Performance comparisons

We compare three groups of portfolio policies: the dynamic portfolio policies, the static portfolio policies with time-varying returns, and the static portfolio policies with time-varying covariances. The static portfolio policy is different from the dynamic portfolio policy in the sense that it only considers one-period mean-variance optmisation, whereas the dynamic portfolio policy considers the evolution of expected returns in all future periods. When incorporating the quadratic transaction costs, the static portfolio policy trades partially towards the expected Markowitz portfolio as opposed to the GP dynamic portfolio policy's aim portfolio that depends on signals' alpha decays. Clearly, this is one shortcoming of the static portfolio policy since the static portfolio policy treats all factors the same, whereas the GP dynamic solution optimises the weights on factors by putting more weight to factors with slower alpha decay.

Table 2 illustrates the performance of various portfolio policies examined by GP and DMN. Panel A replicates GP's result. By following their dynamic trading strategy, we discover that several features mirrored in our commodity futures data are highly consistent with those documented in GP. In particular, we confirm the robustness of the 12-month momentum and 5-year reversal as documented by Erb and Harvey (2006), Asness et al. (2013), and GP. We also achieve very similar mean reversion speeds, which in our case are -0.2423 for a 5-day return predictor, -0.0048 for a 1-year return predictor, and -0.0014 for a 5-year return predictor.

[14] We retain the default values set by Dr. Kevin Sheppard in his MFE Toolbox. The online link for this toolbox is provided at https://www.kevinsheppard.com/MFE_Toolbox.

Insert Table 2 here

The purpose of the comparison in Panel A is to highlight the superior performance of the GP dynamic strategy (Equation 6) in comparison with the Markowitz portfolio (Equation 10) and the static optimisation (Equation 12). Despite using different datasets and focusing on a different time span, we find that the main patterns reflected in both of our datasets are highly consistent with the findings of GP.

Specifically, the Markowitz portfolio has significantly higher gross performance ratio than the GP dynamic strategy. However, the GP dynamic strategy achieves the highest net Sharpe ratio (0.3789), the highest net omega ratio (1.0785), and the highest net reward-to-risk ratio (0.0167) among all the trading strategies in Panel A. By varying the weight from 1% to 10%, we attempt to place a similar weight on the expected Markowitz portfolio in the static portfolio as in the GP dynamic strategy. The varying weights also provide the best possible combination of λ and γ for the static portfolio according to GP. Nevertheless, consistent with the findings of GP, we observe that even placing the best weight on the predicted Markowitz portfolio (= 1%) generates a significantly lower net Sharpe ratio (0.2579), a lower net omega ratio (1.0550), and a lower reward-to-risk ratio (0.0117) than is generated using the GP dynamic strategy. For commodity futures, the GP dynamic strategy outperforms the best static portfolio by approximately 12% in terms of the net Sharpe ratio. The static strategy with a 10% weight has a slightly higher gross Sharpe ratio than the GP dynamic strategy. We observe identical patterns in terms of these two features in GP. Furthermore, in the last row, we observe the performance of the actual static optimisation framework, where the Sharpe, omega, and reward-to-risk ratios before and after transaction costs are much lower than those of the GP dynamic strategy.

The dominances noted above seem to be more obvious when we use ETF data. Although the Markowitz portfolio generates the highest gross Sharpe ratio, which is approximately three times as large as that of the GP dynamic strategy, it delivers negative net Sharpe and reward-to-risk ratios and an almost zero net omega ratio. Similarly, when the weight is changed from 1% to 10%, the gross Sharpe, omega, and reward-to-risk ratios related to the static portfolio are all higher than those of the GP dynamic strategy. The net Sharpe ratios, however, fall below zero once the transaction costs are taken into consideration. The GP dynamic strategy performs significantly better than the majority of the strategies in the presence of transaction costs. GP explain this superiority by examining the different treatments on predicting returns in the dynamic strategy. They state that the dynamic solution places more weights on factors with a slower mean-reversion speed. By contrast, the static solution treats all predicting variables the same.

Panel B reports the performance of the DMN portfolio policy. We find that its performance ratios are numerically higher than any static portfolio shown under both commodity futures and country ETFs. Its drop in either of the performance ratio is not outstanding after transaction costs are considered.

4.2 Portfolio turnover

Figure 1 indicates the position of each commodity futures across the sample periods in the Markowitz and GP dynamic portfolios. Figure 2 indicates the position movement of each country ETF in the Markowitz and GP dynamic portfolios. Consistent with the findings by GP, in both figures, the GP dynamic strategy (labelled as a solid red line) leads to much smoother position variation across the sample period than does the Markowitz portfolio (labelled as a dashed blue line). Meanwhile, the GP dynamic strategy captures the excess price changes because it produces a position trend that is identical to that of the Markowitz portfolio in both datasets. Indeed, it yields lower transaction costs than the Markowitz portfolio without sacrificing much the abilities to pursue gains. Taking the position of Copper, for example, the Markowitz and dynamic curves both climb to their peaks before dropping dramatically, and they subsequently experience a gradual rise, followed by a marginal fall at the end of the sample period.

Insert Figures 1 and 2 here

4.3 Robust target portfolios

Panel A of Table 3 presents the performance of static mean-variance strategies with various time-varying covariance forecasts. In Panel A, each risk strategy delivers a very similar portfolio performance both before and after transaction costs. Although the Markowitz portfolios with time-varying covariances achieve higher gross Sharpe, omega, and reward-to-risk ratios than those of the static optimisations before transaction costs, their net performance is significantly worse than those of the static optimisations with time-varying covariances. Indeed, compared with the GP dynamic strategy, each static optimisation with time-varying covariances generate competitive net performance. Although the difference in the Sharpe ratio is not statistically significant, each static portfolio policy with time-varying covariances yields numerically higher net Sharpe, omega, and reward-to-risk ratios and higher gross Sharpe, omega, and reward-to-risk ratios than those of the GP dynamic strategy. The net Sharpe ratio from each static optimisation is approximately 20% higher and 15% higher than that of the GP dynamic strategy when the commodity futures and ETF data are used, respectively. However, the findings are different when we recall the static optimisation with time-varying returns in the previous table. Different from the previous portfolio policies with predictable returns, the existing portfolio trades towards the target portfolio with time-varying covariances in each period. Importantly, the gap between the gross and net performance ratios is marginal for the static optimisation with time-varying covariances, indicating moderate position variations across periods.

Insert Table 3 here

Interestingly, the higher performance ratios of the static optimisation with time-varying covariances over the GP dynamic strategy with predictable returns is in contrast to the predictability patterns demonstrated by the performance of the Markowitz portfolios. Using ETF data, we find that each Markowitz portfolio with time-varying covariances contributes to *lower* gross Sharpe, omega, and reward-to-risk ratios than those of the Markowitz portfolio with time-varying returns. The predictability in input estimators clearly does not play a role in the higher performance ratios of the static optimisation with time-varying covariances over the GP dynamic strategy (with time-varying returns).

Regarding the comparison with the DMN portfolio policy, there is no evidence that the DMN portfolio policy has advantages over the static optimisation with time-varying covariances in terms of either the net performance or the reduction in trading costs. Although the difference in the Sharpe ratio is not statistically significant, the static optimisation with time-varying covariances has numerically higher Sharpe, omega, and reward-to-risk ratios than the DMN portfolio policy both before and after transaction costs with the commodity futures data. The gap between gross and net performance ratios is smaller for the static optimisation with time-varying covariances than for the DMN portfolio policy with the ETF data. Apparently, a target portfolio with time-varying covariances is able to capture price change gains with only moderate updates on asset positions.

Panel B of Table 3 demonstrates the performances of the short sale-constrained and norm-constrained static portfolios. We place restrictions on asset positions in the static portfolio with time-varying returns. Although the single-period static strategy ignores the entire picture when the asset positions are rebalanced, we find that both the short sale-constrained and the norm-constrained portfolios exhibit somewhat competitive performance levels. Because of the restriction on position movement, the gap between the gross and net performance ratios is very small. Interestingly, the norm-constrained portfolio has a higher gross Sharpe/omega/reward-to-risk ratio and a higher net Sharpe/omega/reward-to-risk ratio than the short sale-constrained portfolio. This finding is in line with the existing literature that stricter restrictions on asset positions/weights, such as stricter upper bounds, significantly improve portfolio performance (see, e.g., Frost and Savarino, 1988; DeMiguel et al., 2009a). When comparing these two constrained portfolio types with the previous static optimisation models, we find that their net Sharpe ratios (0.4410 and 0.4938 for commodity futures and 0.2915 and 0.3767 for country ETFs), net omega ratios (1.1002 and 1.1116 for commodity futures and 1.0577 and 1.0743 for country ETFs), and net reward-to-risk ratios

(0.0177 and 0.0194 for commodity futures and 0.0115 and 0.0146 for country ETFs) are second only to the static optimisation with volatility timing. Although their gross Sharpe, omega, and reward-to-risk ratios are lower than those of either of the Markowitz portfolios, they produce much higher net performance ratios.

Regarding the comparison with the GP dynamic portfolio policy, the two constrained portfolios lead to higher net Sharpe, omega, and reward-to-risk ratios with the difference in the Sharpe ratio being statistically insignificant. The pattern is slightly different in the horse race with the DMN portfolio policy. The better performance of the constrained portfolios over the DMN portfolio policy applies to commodity futures data but becomes weak when ETF data are used.

One major reason for the good performance of our static portfolio policies lies in the robust target portfolio generated when either time-varying covariances or constrained trading positions are applied. That is, the static optimisation is able to perform as well as the GP and DMN dynamic portfolio policies once the current static portfolio trades towards a more robust target portfolio. Although GP critises that the static portfolio policy does not optimise the weights on different factors and ignore all future events, the robust target portfolio in the static portfolio policy would dramatically improve portfolio performance. In addition, the low weight imposed on the predicted Markowitz portfolio (Equation 12) results in a low scale of transaction costs for the static optimisation. In each period, the existing portfolio moves very smoothly towards the target portfolio (the predicted Markowitz portfolio) producing low transaction costs.

To further look at the importance of the robust target portfolio, we allow both the time-varying covariances and the short-sale constraints in the static portfolio policy. The results are shown in Table 4. Because the model-implied weight on the target portfolio remains unchanged, the short sale-constrained static portfolio policies with time-varying covariances generate low level of transaction costs that are very similar to those of any static portfolio policy in previous tables. However, with the robust target portfolio, the short sale-constrained static portfolio policies with time-varying covariances perform much better than the static portfolio policies with time-varying returns. In terms of the comparison with the GP dynamic portfolio policy, these static portfolio policies yield higher net performance ratios with the commodity futures (e.g. 0.6013 *vs.* 0.3789) and similar performance ratios with the country ETFs (e.g. 0.2084 *vs.* 0.2639). The differences in the Sharpe ratios are, however, not statistically significant in all cases. Regarding the comparison with the static optimisation with either time-varying covariances or constrained portfolio weights, no obvious improvement is observed in terms of the empirical performance.

Insert Table 4 here

4.4 Simulation results

To further study the properties of each portfolio policy, we simulate two datasets with 25 and 50 risky assets, respectively. We follow the simulation approach in DeMiguel et al. (2015). Specifically, we assume that price changes are from a multivariate normal distribution. The annual mean price changes are uniformly distributed in the interval [0.05, 0.12]. The covariance matrix of price changes is a diagonal matrix with its variance elements drawn from a uniform distribution in the interval [0.1, 0.5]. The parameters λ, γ, and ρ are consistent with their values in GP. Our sample period is set as 20 years with 5200 (20×260) observations. We use Monte Carlo sampling to generate price changes and then compute the average performance ratios of all paths. The results are reported in Table 5. Consistent with the findings from the empirical results, we find that both the static optimisation with time-varying covariances[15] and the short sale-constrained static portfolio policy outperform the GP dynamic optimisation in both $N = 25$ and $N = 50$ cases. In particular, these two approaches with the robust target portfolios generate higher average performance ratios than those of the GP dynamic optimisation both before and after transaction costs are considered. For example, in the case of $N = 25$, the static optimisation with time-varying covariances produce higher

[15]Because the static optimisation with all six risk forecasting techniques produce very similar results, we use the RM case to represent the whole group.

average gross Sharp ratio (0.8430 > 0.2489), omega ratio (1.1458 > 1.0413), and reward-to-risk ratio (0.0326 > 0.0097) and higher average net Sharpe ratio (0.8424 > 0.2098), omega ratio (1.1457 > 1.0349), and reward-to-risk ratio (0.0325 > 0.0082) than those of the GP dynamic portfolio policy. The gap between the average gross and net performance ratios is also lower in the case of the static optimisation (about 0.002 drop in the Sharpe ratio) than that of the GP dynamic optimisation (about 0.04 drop in the Sharpe ratio).

Insert Table 5 here

4.5 Robustness checks

To gauge the robustness of our findings, we check the sensitivity of the portfolio performance to the key base parameters and the covariance matrix. First, we examine the sensitivity of portfolio performance in all analyses to the key parameter settings. Here, we apply three different sets of trading parameters. Table 6 concerns a low transaction cost scenario wherein $\lambda^{LOW} = \frac{\lambda}{2}$. Apart from the direct influence on transaction costs, lowering the amount of λ leads to more aggressive trading behaviour towards the target portfolio because the weight placed on the current portfolio x_{t-1} falls. By contrast, a relatively high transaction cost scenario wherein $\lambda^{HIGH} = 2\lambda$ in Table 7 results in a conservative moving trajectory towards the target. The findings in both tables indicate that the competitive performance of the static portfolio with either time-varying covariances or the norm constraints over the GP-type portfolio policy is insensitive to the change in λ. In particular, when we have relatively low transaction costs, each static portfolio policy with time-varying covariances generates a higher omega ratio than does the DMN portfolio policy before and after transaction costs.

Insert Tables 6, 7, and 8 here

To show the validity of our results, we also apply the parameters adopted by GP to commodity futures data, as shown in Table 8. To conform to their parameters, we set

λ to 5×10^{-7} and γ to 10^{-9}. We find that the influences on the performance ratio both before and after transaction costs are somewhat marginal. This time, the static optimisation with time-varying covariances, the short sale-constrained portfolio, and the norm-constrained portfolio all yield higher performance ratios than either the GP dynamic strategy or the DMN portfolio policy with and without transaction costs with the difference in the Sharpe ratio being statistically insignificant.

Second, we substantially improve the predictability in the case of time-varying returns for the portfolio of commodity futures. Here, we are attempting to prove that the predictive ability associated with the model inputs is irrelevant with respect to the competitive performance of the static portfolio over the dynamic portfolio policy. This conclusion is clearly documented in the case of ETFs, given that the Markowitz portfolio associated with time-varying returns generates higher gross Sharpe ratios than the Markowitz portfolio with volatility timing. Although time-varying returns are able to capture more price movements, the volatility timing static optimisation, which has the disadvantage on model inputs, still performs, at least, as well as the GP-type portfolio policy. However, this finding is not demonstrated based on commodity futures data because the Markowitz portfolio associated with time-varying returns performs worse than the Markowitz portfolio with time-varying covariances before transaction costs. The better performance of the static portfolio policy with time-varying covariances over the dynamic portfolio policy may be observed because the risk forecast contains less estimation error than the return forecast.

By not shrinking the covariance matrix, we are able to achieve a higher level of predictability in the combination of varying returns and fixed risk, as shown by the relative performances of the two Markowitz portfolios in Panels B and C of Table 9. Even with the estimation disadvantages, each static optimisation with time-varying covariances in Panel C yields statistically insignificantly higher Sharpe ratio than the two GP-type frameworks in Panel A both before and after transaction costs. Similar findings are found in shrinking the fixed sample covariance matrix towards (1) a constant-correlation matrix and (2) a one-parameter matrix. The results are demonstrated in Tables 10 and 11, respectively. Interestingly, in all three cases, the norm-constrained portfolio constantly yields a higher Sharpe/omega ratio than the two dynamic optimal portfolio solutions in the presence/absence of transaction costs.

Insert Tables 9, 10, and 11 here

5 Conclusions

By using both empirical and simulated data, we find no evidence that the dynamic optimal portfolio policies are superior to the static portfolio policies that trade towards the robust target portfolio. Specifically, the static portfolio policy with variation in the covariance matrix performs at least on par with the GP and DMN dynamic portfolio policies both before and after transaction costs. Meanwhile, it generates lower transaction costs throughout the holding period. The competitive performance is demonstrated by the static portfolio policy under a variety of risk forecasting techniques. We also examine the use of a position constraint in the static portfolio policy that accounts for transaction costs. Apparently, for all three datasets, the short sale-constrained portfolio with returns estimated by the same technique as the GP dynamic strategy performs as well as the GP dynamic strategy both before and after transaction costs. Similar performance patterns are also found in the comparison with another dynamic optimal portfolio policy developed by DMN. For both commodity futures and ETF data, we show that the predictability of the model inputs is irrelevant with respect to the competitive performance of the static portfolio policy over the dynamic portfolio policy. Indeed, the good performance of the static portfolio policy holds even when the model inputs in the dynamic portfolio policy have a higher degree of forecastability. In addition, we show that the trading parameters (λ and γ) are not the main drivers of relative performance. We hope that our work in this paper can play an active role in encouraging the incorporation of time-varying covariances and norm bounding into future studies of dynamic trading strategies.

Table 1: Summary statistics

This table reports the average asset price, the annualised return volatility, and the average daily trading volume for both commodity futures and country ETFs. Panel A shows the summary statistics of commodity futures. Panel B shows the summary statistics of ETFs.

	Average price per asset	Annualised return volatility	Contract multiplier	Average daily trading volume
Panel A: Commodity futures				
Brent Crude Oil	57966	0.3609	1000	118956
Cocoa	18690	0.3391	10	43080
Coffee	42470	0.4004	37500	43954
Copper	50033	0.3039	25000	19546
Gasoil	50356	0.3483	100	54116
Gold	69703	0.2065	100	120534
Heating Oil	68082	0.4012	42000	41691
Natgas	51029	0.6267	10000	77566
Platinum	50080	0.3338	50	6007
Sugar	13615	0.4050	112000	176763
WTI Crude Oil	56599	0.4093	1000	173402
Panel B: Country ETFs				
Canada	22.5594	0.2409		1295818
France	24.3028	0.2925		272851
Germany	21.4375	0.2535		1682913
Italy	19.9423	0.2436		466034
Japan	10.5121	0.3028		19061351
UK	17.4842	0.2850		960803

Table 2: Performance of portfolio policies examined by GP and DMN

This table shows the annualised gross Sharpe ratio, the gross omega ratio, the gross mean-value-at-risk ratio, the annualised net Sharpe ratio, the net omega ratio, and the net mean-value-at-risk ratio for both dynamic and static portfolio policies. 'Gross' means before transaction costs and 'Net' means after transaction costs. 'SR' stands for the Sharpe ratio. 'OR' stands for the omega ratio. 'RR' stands for the reward-to-risk ratio (mean-value-at-risk ratio). 'Markowitz' is the classical Markowitz portfolio (Equation 10). 'GP dynamic optimisation' is the dynamic portfolio policy developed by GP (Equation 6). 'Static portfolio' is the Markowitz portfolio accounting for transaction costs with varying weights on the expected Markowitz portfolio (Equation 12). To illustrate the superior performance of the GP dynamic optimisation, we allow the weight on the target portfolio (expected Markowitz portfolio) in the static portfolio to vary from 1% to 10%, which is an approach employed by GP. 'Static optimisation' is the static mean-variance strategy considering transaction costs with the model-implied weight on the expected Markowitz portfolio (Equation 12). 'DMN portfolio policy' is the dynamic portfolio policy developed by DMN (Equations 7 and 9). The *p*-value for the Sharpe ratio difference between each portfolio policy and the GP dynamic portfolio policy is shown in parentheses.

	Gross SR	Net SR	Gross OR	Net OR	Gross RR	Net RR
Panel A: Commodity futures						
GP dynamic optimisation	0.4047	0.3789	1.0841	1.0785	0.0178	0.0167
DMN portfolio policy	0.2875	0.2864	1.0565	1.0562	0.0110	0.0109
	(0.8102)	(0.8472)				
Markowitz	0.6315	–13.8111	1.1373	0.0022	0.0293	–0.3775
	(0.0879)	(0.1279)				
Static portfolio						
Weight on Markowitz=10%	0.4225	–2.0133	1.0946	0.6576	0.0203	–0.0833
	(0.9191)	(0.0010)				
Weight on Markowitz=9%	0.4141	–1.6043	1.0929	0.7142	0.0199	–0.0688
	(0.9510)	(0.0010)				
Weight on Markowitz=8%	0.4048	–1.2269	1.0909	0.7716	0.0195	–0.0540
	(0.9970)	(0.0010)				
Weight on Markowitz=7%	0.3945	–0.8842	1.0886	0.8285	0.0189	–0.0395
	(0.9381)	(0.0010)				
Weight on Markowitz=6%	0.3831	–0.5796	1.0860	0.8835	0.0184	–0.0264
	(0.8531)	(0.0010)				
Weight on Markowitz=5%	0.3707	–0.3159	1.0830	0.9345	0.0178	–0.0146
	(0.7702)	(0.0010)				
Weight on Markowitz=4%	0.3575	–0.0956	1.0798	0.9797	0.0171	–0.0045
	(0.6284)	(0.0010)				
Weight on Markowitz=3%	0.3438	0.0787	1.0763	1.0170	0.0165	0.0037
	(0.5105)	(0.0050)				
Weight on Markowitz=2%	0.3279	0.2030	1.0720	1.0440	0.0158	0.0097
	(0.3117)	(0.0420)				
Weight on Markowitz=1%	0.2929	0.2579	1.0627	1.0550	0.0133	0.0117
	(0.0969)	(0.0659)				
Static optimisation	0.1710	0.1696	1.0344	1.0341	0.0072	0.0071
	(0.0959)	(0.1399)				

Table 2 (cont.)

	Gross SR	Net SR	Gross OR	Net OR	Gross RR	Net RR
Panel B: Country ETFs						
GP dynamic optimisation	0.3889	0.2639	1.0842	1.0564	0.0184	0.0124
DMN portfolio policy	0.4680	0.4628	1.0811	1.0802	0.0185	0.0183
	(0.8561)	(0.6394)				
Markowitz	1.0684	–5.1092	1.3455	0.0000	0.0699	–0.3217
	(0.1249)	(0.0020)				
Static portfolio						
Weight on Markowitz=10%	1.1320	–7.5997	1.1320	0.0112	0.0294	–0.3089
	(0.1389)	(0.0010)				
Weight on Markowitz=9%	0.6266	–7.5485	1.1282	0.0163	0.0283	–0.3065
	(0.1479)	(0.0020)				
Weight on Markowitz=8%	0.6150	–7.4826	1.1244	0.0239	0.0272	–0.3069
	(0.1618)	(0.0030)				
Weight on Markowitz=7%	0.6016	–7.3907	1.1204	0.0365	0.0261	–0.3047
	(0.1968)	(0.0010)				
Weight on Markowitz=6%	0.5858	–7.2476	1.1160	0.0583	0.0252	–0.3009
	(0.2198)	(0.0010)				
Weight on Markowitz=5%	0.5672	–6.9920	1.1114	0.0974	0.0238	–0.2911
	(0.2677)	(0.0010)				
Weight on Markowitz=4%	0.5458	–6.4680	1.1065	0.1694	0.0230	–0.2561
	(0.2717)	(0.0010)				
Weight on Markowitz=3%	0.5225	–5.3040	1.1018	0.3046	0.0217	–0.1966
	(0.3746)	(0.0010)				
Weight on Markowitz=2%	0.5005	–3.0586	1.0980	0.5505	0.0210	–0.1133
	(0.3666)	(0.0010)				
Weight on Markowitz=1%	0.4761	–0.5632	1.0975	0.8955	0.0219	–0.0250
	(0.5195)	(0.0010)				
Static optimisation	0.1361	0.1281	1.0239	1.0225	0.0054	0.0051
	(0.3177)	(0.5964)				

Table 3: Performance of static portfolio policies with time-varying covariances and norm-constrained static portfolio policies

This table illustrates the annualised gross Sharpe ratio, the gross omega ratio, the gross mean-value-at-risk ratio, the net annualised Sharpe ratio, the net omega ratio, and the net mean-value-at-risk ratio for the static portfolio strategy with either time-varying covariances or norm constraints. 'Gross' means before transaction costs, and 'Net' means after transaction costs. 'SR' stands for the Sharpe ratio. 'OR' stands for the omega ratio. 'RR' stands for the reward-to-risk ratio (mean-value-at-risk ratio). 'RM' represents the exponentially weighted moving average model. 'RM2006' represents a model that is the weighted average of RMs. 'CCC-GJR' represents the constant conditional correlation multivariate volatility model with univariate volatilities estimated by the GJR-GARCH model. 'DCC-GJR' represents the dynamic conditional correlation multivariate volatility model with univariate volatilities estimated by the GJR-GARCH model. 'RBEKK' represents the rotated BEKK multivariate volatility model. 'RDCC-GJR' represents the rotated DCC multivariate volatility model with univariate volatilities estimated by the GJR-GARCH model. 'Markowitz' is the classical Markowitz portfolio (Equation 10). 'Static optimisation' is the static mean-variance strategy considering transaction costs with the model-implied weight on the expected Markowitz portfolio (Equation 12). The p-value for the Sharpe ratio difference between each portfolio policy and the GP dynamic portfolio policy is shown in parentheses. For those net RRs that have the same numerical values as the gross RRs, the difference appears from the sixth digit, which is not shown here.

		Gross SR	Net SR	Gross OR	Net OR	Gross RR	Net RR
Panel A: Commodity futures							
RM	Markowitz	0.8940	−2.0388	1.2032	0.0065	0.0411	−0.4341
		(0.2977)	(0.0010)				
	Static optimisation	0.6154	0.6060	1.1266	1.1245	0.0248	0.0244
		(0.6494)	(0.6184)				
RM2006	Markowitz	0.8998	−2.5035	1.1981	0.0066	0.0404	−0.3580
		(0.2228)	(0.0010)				
	Static optimisation	0.6064	0.5995	1.1246	1.1230	0.0243	0.0240
		(0.6663)	(0.5934)				
CCC-GJR	Markowitz	0.9698	−1.5940	1.1959	0.0571	0.0422	−0.4927
		(0.1489)	(0.0010)				
	Static optimisation	0.5893	0.5844	1.1197	1.1187	0.0234	0.0232
		(0.6763)	(0.6853)				
DCC-GJR	Markowitz	0.9186	−1.7376	1.1884	0.0643	0.0403	−0.4142
		(0.1968)	(0.0010)				
	Static optimisation	0.5912	0.5867	1.1212	1.1202	0.0235	0.0234
		(0.6893)	(0.6593)				
RBEKK	Markowitz	0.9476	−0.3408	1.1824	0.2614	0.0385	−0.4531
		(0.1938)	(0.2717)				
	Static optimisation	0.5741	0.5722	1.1152	1.1148	0.0228	0.0228
		(0.7103)	(0.6703)				
RDCC-GJR	Markowitz	0.9218	−1.7081	1.1891	0.0658	0.0402	−0.4174
		(0.2018)	(0.0010)				
	Static optimisation	0.5907	0.5863	1.1211	1.1201	0.0235	0.0233
		(0.6973)	(0.6693)				
Short sale-constrained portfolio		0.4444	0.4410	1.1011	1.1002	0.0179	0.0177
		(0.9201)	(0.8591)				
Norm-constrained portfolio		0.4976	0.4938	1.1126	1.1116	0.0196	0.0194
		(0.7702)	(0.7343)				

Table 3 (cont.)

		Gross SR	Net SR	Gross OR	Net OR	Gross RR	Net RR
Panel B: Country ETFs							
RM	Markowitz	0.8404	–2.4667	1.1611	0.0305	0.0331	–0.3203
		(0.1918)	(0.3946)				
	Static optimisation	0.4261	0.4254	1.0823	1.0822	0.0164	0.0164
		(0.9351)	(0.6963)				
RM2006	Markowitz	0.8648	–2.6996	1.1650	0.0214	0.0341	–0.3549
		(0.1658)	(0.0480)				
	Static optimisation	0.4173	0.4167	1.0809	1.0808	0.0161	0.0161
		(0.9331)	(0.7373)				
CCC-GJR	Markowitz	0.8016	–2.1355	1.1516	0.0702	0.0309	–0.2933
		(0.2218)	(0.0020)				
	Static optimisation	0.4331	0.4324	1.0834	1.0833	0.0166	0.0166
		(0.9261)	(0.7203)				
DCC-GJR	Markowitz	0.8441	–2.3868	1.1565	0.0714	0.0327	–0.3104
		(0.2028)	(0.0120)				
	Static optimisation	0.4284	0.4278	1.0825	1.0824	0.0163	0.0162
		(0.9361)	(0.7393)				
RBEKK	Markowitz	0.7041	–0.4618	1.1254	0.4123	0.0274	–0.2157
		(0.4126)	(0.3387)				
	Static optimisation	0.4558	0.4554	1.0871	1.0870	0.0175	0.0175
		(0.8831)	(0.6643)				
RDCC-GJR	Markowitz	0.8401	–2.3935	1.1557	0.0709	0.0327	–0.3145
		(0.1988)	(0.0020)				
	Static optimisation	0.4286	0.4281	1.0826	1.0825	0.0163	0.0162
		(0.9221)	(0.7443)				
Short sale-constrained portfolio		0.3002	0.2915	1.0595	1.0577	0.0118	0.0115
		(0.7732)	(0.9201)				
Norm-constrained portfolio		0.3860	0.3767	1.0762	1.0743	0.0150	0.0146
		(0.9860)	(0.6793)				

Table 4: Performance of short sale-constrained static portfolio policies with time-varying covariances

This table illustrates the annualised gross Sharpe ratio, the gross omega ratio, the gross mean-value-at-risk ratio, the net annualised Sharpe ratio, the net omega ratio, and the net mean-value-at-risk ratio for the short sale-constrained static portfolio strategy with time-varying covariances. 'Gross' means before transaction costs, and 'Net' means after transaction costs. 'SR' stands for the Sharpe ratio. 'OR' stands for the omega ratio. 'RR' stands for the reward-to-risk ratio (mean-value-at-risk ratio). 'RM' represents the exponentially weighted moving average model. 'RM2006' represents a model that is the weighted average of RMs. 'CCC-GJR' represents the constant conditional correlation multivariate volatility model with univariate volatilities estimated by the GJR-GARCH model. 'DCC-GJR' represents the dynamic conditional correlation multivariate volatility model with univariate volatilities estimated by the GJR-GARCH model. 'RBEKK' represents the rotated BEKK multivariate volatility model. 'RDCC-GJR' represents the rotated DCC multivariate volatility model with univariate volatilities estimated by the GJR-GARCH model. 'Static optimisation' is the static mean-variance strategy considering transaction costs with the model-implied weight on the expected Markowitz portfolio (Equation 12). The *p*-value for the Sharpe ratio difference between each portfolio policy and the GP dynamic portfolio policy is shown in parentheses. For those net RRs that have the same numerical values as the gross RRs, the difference appears from the sixth digit, which is not shown here.

	Gross SR	Net SR	Gross OR	Net OR	Gross RR	Net RR
Panel A: Commodity futures						
Static optimisation (RM)	0.6115	0.6013	1.1257	1.1234	0.0249	0.0245
	(0.6533)	(0.6494)				
Static optimisation (RM2006)	0.6045	0.5975	1.1241	1.1226	0.0244	0.0241
	(0.6753)	(0.6334)				
Static optimisation (CCC-GJR)	0.5882	0.5833	1.1197	1.1186	0.0234	0.0232
	(0.6733)	(0.6523)				
Static optimisation (DCC-GJR)	0.5896	0.5851	1.1209	1.1199	0.0236	0.0235
	(0.7023)	(0.6404)				
Static optimisation (RBEKK)	0.5708	0.5689	1.1144	1.1140	0.0227	0.0227
	(0.7323)	(0.7013)				
Static optimisation (RDCC-GJR)	0.5890	0.5845	1.1208	1.1199	0.0236	0.0234
	(0.6903)	(0.6743)				
Panel B: Country ETFs						
Static optimisation (RM)	0.2088	0.2084	1.0413	1.0412	0.0082	0.0082
	(0.6973)	(0.8911)				
Static optimisation (RM2006)	0.2043	0.2039	1.0404	1.0403	0.0080	0.0080
	(0.6693)	(0.8981)				
Static optimisation (CCC-GJR)	0.2050	0.2046	1.0404	1.0403	0.0080	0.0080
	(0.6643)	(0.8851)				
Static optimisation (DCC-GJR)	0.2054	0.2050	1.0405	1.0404	0.0080	0.0080
	(0.6523)	(0.8961)				
Static optimisation (RBEKK)	0.1938	0.1936	1.0382	1.0382	0.0074	0.0074
	(0.6474)	(0.8921)				
Static optimisation (RDCC-GJR)	0.2054	0.2050	1.0404	1.0404	0.0080	0.0080
	(0.6703)	(0.8831)				

Table 5: Portfolio performance based on simulated price changes

This table shows the average annualised gross Sharpe ratio, the average gross omega ratio, the average gross mean-value-at-risk ratio, the average net annualised Sharpe ratio, the average net omega ratio, and the average net mean-value-at-risk ratio of various portfolio policies based on simulated data. We follow the simulation approach in DeMiguel et al. (2015) and simulate two datasets with 25 and 50 risky assets, respectively. We use Monte Carlo sampling to simulate price changes and in each path, we simulate 5200 observations. The portfolio policies examined include one dynamic portfolio policy, three static portfolio policies with time-varying returns, and two static portfolio policies with time-varying covariances.

		Gross SR	Net SR	Gross OR	Net OR	Gross RR	Net RR
Panel A: $N = 25$							
Dynamic portfolio policies							
GP dynamic optimisation		0.2489	0.2098	1.0413	1.0349	0.0097	0.0082
Static portfolio policies with time-varying returns							
Markowitz		0.3473	–16.9505	1.0574	0.2367	0.0135	–0.3366
Static optimisation		0.1421	0.1403	1.0252	1.0249	0.0056	0.0055
Short sale-constrained portfolio		0.4726	0.3348	1.0818	1.0575	0.0176	0.0110
Static portfolio policies with time-varying covariances							
RM	Markowitz	0.8057	–6.2251	1.1352	0.3490	0.0315	–0.1922
	Static optimisation	0.8430	0.8424	1.1458	1.1457	0.0326	0.0325
Panel B: $N = 50$							
Dynamic portfolio policies							
GP dynamic optimisation		0.2694	0.2300	1.0452	1.0388	0.0105	0.0090
Static portfolio policies with time-varying returns							
Markowitz		0.3677	–21.1315	1.0608	0.2353	0.0143	–0.3695
Static optimisation		0.1695	0.1677	1.0309	1.0306	0.0068	0.0067
Short sale-constrained portfolio		0.6068	0.4901	1.1056	1.0851	0.0220	0.0157
Static portfolio policies with time-varying covariances							
RM	Markowitz	1.1182	–8.8022	1.1916	0.2048	0.0442	–0.2473
	Static optimisation	1.1908	1.1898	1.2115	1.2113	0.0467	0.0466

Table 6: Portfolio performance when transaction costs are low

This table shows the performance of various portfolio policies using both commodity futures and country ETFs. Half of the transaction costs assumed in the baseline analysis are considered. The portfolio policies shown include two dynamic portfolio policies, two static mean-variance portfolio policies with time-varying returns, two norm-constrained mean-variance portfolio policies, and 12 static mean-variance portfolio policies with time-varying covariances. The p-value for the Sharpe ratio difference between each portfolio policy and the GP dynamic portfolio policy is shown in parentheses. For those net RRs that have the same numerical values as the gross RRs, the difference appears from the sixth digit, which is not shown here.

		Gross SR	Net SR	Gross OR	Net OR	Gross RR	Net RR
Panel A: Commodity futures							
Dynamic portfolio policies							
GP dynamic optimisation		0.4045	0.3643	1.0841	1.0754	0.0178	0.0160
DMN portfolio policy		0.2860	0.2851	1.0561	1.0559	0.0110	0.0110
		(0.8052)	(0.8711)				
Static portfolio policies with time-varying returns							
Markowitz		0.6315	–13.5857	1.1373	0.0097	0.0293	–0.3726
		(0.0360)	(0.0779)				
Static optimisation		0.1840	0.1821	1.0375	1.0372	0.0079	0.0078
		(0.0460)	(0.0829)				
Short sale-constrained		0.4915	0.4864	1.1166	1.1153	0.0211	0.0209
		(0.5095)	(0.5036)				
Norm-constrained		0.5095	0.5036	1.1150	1.1136	0.0206	0.0204
		(0.7632)	(0.6703)				
Static portfolio policies with time-varying covariances							
RM	Markowitz	0.8940	–2.0295	1.2032	0.0176	0.0411	–0.4259
		(0.2607)	(0.0010)				
	Static optimisation	0.7226	0.7050	1.1527	1.1488	0.0301	0.0294
		(0.5085)	(0.4316)				
RM2006	Markowitz	0.8998	–2.4906	1.1981	0.0181	0.0404	–0.3535
		(0.2158)	(0.0010)				
	Static optimisation	0.7173	0.7048	1.1495	1.1467	0.0293	0.0287
		(0.4515)	(0.4635)				
CCC-GJR	Markowitz	0.9698	–1.5630	1.1959	0.1341	0.0422	–0.4163
		(0.1508)	(0.0010)				
	Static optimisation	0.6982	0.6893	1.1400	1.1381	0.0283	0.0279
		(0.5075)	(0.4905)				
DCC-GJR	Markowitz	0.9186	–1.6976	1.1884	0.1522	0.0403	–0.3560
		(0.2058)	(0.0010)				
	Static optimisation	0.7060	0.6982	1.1435	1.1418	0.0286	0.0282
		(0.5065)	(0.4525)				
RBEKK	Markowitz	0.9476	–0.3192	1.1824	0.4454	0.0385	–0.2434
		(0.1928)	(0.0569)				
	Static optimisation	0.6842	0.6812	1.1354	1.1348	0.0269	0.0268
		(0.5445)	(0.4805)				
RDCC-GJR	Markowitz	0.9218	–1.6683	1.1891	0.1548	0.0402	–0.3474
		(0.2098)	(0.0010)				
	Static optimisation	0.7049	0.6971	1.1433	1.1416	0.0285	0.0282
		(0.5085)	(0.4815)				

Table 6 (cont.)

		Gross SR	Net SR	Gross OR	Net OR	Gross RR	Net RR
Panel B: Country ETFs							
Dynamic portfolio policies							
GP dynamic optimisation		0.3808	0.2750	1.0818	1.0585	0.0177	0.0127
DMN portfolio policy		0.4750	0.4713	1.0822	1.0816	1.0822	1.0816
		(0.8092)	(0.6174)				
Static portfolio policies with time-varying returns							
Markowitz		1.0684	–5.1083	1.3455	0.0000	0.0699	–0.3214
		(0.4406)	(0.4456)				
Static optimisation		0.1968	0.1874	1.0351	1.0334	0.0078	0.0074
		(0.7213)	(0.7423)				
Short sale-constrained		0.3779	0.3677	1.0779	1.0757	0.0152	0.0148
		(0.9910)	(0.7453)				
Norm-constrained		0.4493	0.4385	1.0905	1.0882	0.0174	0.0170
		(0.7942)	(0.5724)				
Static portfolio policies with time-varying covariances							
RM	Markowitz	0.8404	–2.4197	1.1611	0.0920	0.0331	–0.2980
		(0.1778)	(0.1948)				
	Static optimisation	0.4700	0.4691	1.0902	1.0900	0.0186	0.0186
		(0.8432)	(0.6663)				
RM2006	Markowitz	0.8648	–2.6550	1.1650	0.0695	0.0341	–0.3254
		(0.1758)	(0.0370)				
	Static optimisation	0.4605	0.4598	1.0888	1.0887	0.0182	0.0182
		(0.8501)	(0.7063)				
CCC-GJR	Markowitz	0.8016	–2.0774	1.1516	0.1722	0.0309	–0.2661
		(0.2468)	(0.0020)				
	Static optimisation	0.4763	0.4755	1.0912	1.0911	0.0187	0.0187
		(0.8352)	(0.6414)				
DCC-GJR	Markowitz	0.8441	–2.3070	1.1565	0.1834	0.0327	–0.2509
		(0.1868)	(0.0020)				
	Static optimisation	0.4723	0.4716	1.0903	1.0902	0.0186	0.0186
		(0.8182)	(0.6733)				
RBEKK	Markowitz	0.7041	–0.4164	1.1254	0.6255	0.0274	–0.1138
		(0.3786)	(0.0340)				
	Static optimisation	0.4832	0.4828	1.0915	1.0914	0.0193	0.0193
		(0.8252)	(0.6154)				
RDCC-GJR	Markowitz	0.8401	–2.3139	1.1557	0.1824	0.0327	–0.2535
		(0.1928)	(0.0030)				
	Static optimisation	0.4725	0.4719	1.0903	1.0902	0.0185	0.0185
		(0.8462)	(0.6573)				

Table 7: Portfolio performance when transaction costs are high

This table shows the performance of various portfolio policies using both commodity futures and country ETFs. We double the transaction costs in the baseline analysis for robustness checks. The portfolio policies shown include two dynamic portfolio policies, two static mean-variance portfolio policies with time-varying returns, two norm-constrained mean-variance portfolio policies, and 12 static mean-variance portfolio policies with time-varying covariances. The p-value for the Sharpe ratio difference between each portfolio policy and the GP dynamic portfolio policy is shown in parentheses. For those net RRs that have the same numerical values as the gross RRs, the difference appears from the sixth digit, which is not shown here.

		Gross SR	Net SR	Gross OR	Net OR	Gross RR	Net RR
Panel A: Commodity futures							
Dynamic portfolio policies							
GP dynamic optimisation		0.3948	0.3702	1.0816	1.0763	0.0172	0.0161
DMN portfolio policy		0.2893	0.2876	1.0569	1.0566	0.0111	0.0110
		(0.8322)	(0.8741)				
Static portfolio policies with time-varying returns							
Markowitz		0.6315	–13.8445	1.1373	0.0003	0.0293	–0.3795
		(0.4605)	(0.5265)				
Static optimisation		0.2094	0.2083	1.0418	1.0416	0.0090	0.0090
		(0.4575)	(0.5385)				
Short sale-constrained		0.3985	0.3959	1.0868	1.0862	0.0155	0.0154
		(0.9880)	(0.9381)				
Norm-constrained		0.4195	0.4168	1.0937	1.0930	0.0165	0.0164
		(0.9680)	(0.9570)				
Static portfolio policies with time-varying covariances							
RM	Markowitz	0.8940	–2.0434	1.2032	0.0023	0.0411	–0.4325
		(0.2837)	(0.0020)				
	Static optimisation	0.4680	0.4628	1.0959	1.0948	0.0182	0.0180
		(0.8831)	(0.8482)				
RM2006	Markowitz	0.8998	–2.5097	1.1981	0.0024	0.0404	–0.3567
		(0.1948)	(0.0010)				
	Static optimisation	0.4632	0.4592	1.0949	1.0941	0.0181	0.0180
		(0.8811)	(0.8472)				
CCC-GJR	Markowitz	0.9698	–1.6089	1.1959	0.0223	0.0422	–0.5653
		(0.1638)	(0.0010)				
	Static optimisation	0.4556	0.4527	1.0934	1.0928	0.0179	0.0177
		(0.9151)	(0.8761)				
DCC-GJR	Markowitz	0.9186	–1.7565	1.1884	0.0249	0.0403	–0.4391
		(0.1848)	(0.0010)				
	Static optimisation	0.4537	0.4510	1.0933	1.0928	0.0179	0.0178
		(0.9181)	(0.8492)				
RBEKK	Markowitz	0.9476	–0.3515	1.1824	0.1349	0.0385	–0.7217
		(0.1898)	(0.3467)				
	Static optimisation	0.4440	0.4428	1.0905	1.0903	0.0176	0.0176
		(0.9081)	(0.8781)				
RDCC-GJR	Markowitz	0.9218	–1.7270	1.1891	0.0255	0.0402	–0.4404
		(0.1818)	(0.0010)				
	Static optimisation	0.4537	0.4510	1.0933	1.0928	0.0179	0.0178
		(0.9101)	(0.8791)				

Table 7 (cont.)

		Gross SR	Net SR	Gross OR	Net OR	Gross RR	Net RR
Panel B: Country ETFs							
Dynamic portfolio policies							
GP dynamic optimisation		0.3833	0.2013	1.0837	1.0431	0.0180	0.0094
DMN portfolio policy		0.4613	0.4540	1.0802	1.0788	0.0182	0.0179
		(0.8342)	(0.5554)				
Static portfolio policies with time-varying returns							
Markowitz		1.0684	–5.1096	1.3455	0.0000	0.0699	–0.3219
		(0.2388)	(0.6863)				
Static optimisation		0.1040	0.0967	1.0182	1.0169	0.0041	0.0038
		(0.2877)	(0.6773)				
Short sale-constrained		0.2362	0.2287	1.0453	1.0438	0.0092	0.0089
		(0.6324)	(0.9111)				
Norm-constrained		0.3315	0.3229	1.0646	1.0629	0.0127	0.0124
		(0.8611)	(0.6683)				
Static portfolio policies with time-varying covariances							
RM	Markowitz	0.8404	–2.4885	1.1611	0.0086	0.0331	–0.3202
		(0.1838)	(0.2498)				
	Static optimisation	0.4042	0.4035	1.0785	1.0783	0.0153	0.0152
		(0.9680)	(0.6484)				
RM2006	Markowitz	0.8648	–2.7203	1.1650	0.0057	0.0341	–0.3583
		(0.1578)	(0.0959)				
	Static optimisation	0.3957	0.3951	1.0771	1.0770	0.0149	0.0149
		(0.9690)	(0.6693)				
CCC-GJR	Markowitz	0.8016	–2.1622	1.1516	0.0227	0.0309	–0.3012
		(0.2388)	(0.2388)				
	Static optimisation	0.4111	0.4105	1.0796	1.0795	0.0154	0.0154
		(0.9341)	(0.6394)				
DCC-GJR	Markowitz	0.8441	–2.4227	1.1565	0.0215	0.0327	–0.3185
		(0.1998)	(0.0140)				
	Static optimisation	0.4063	0.4057	1.0787	1.0786	0.0153	0.0152
		(0.9660)	(0.6523)				
RBEKK	Markowitz	0.7041	–0.4839	1.1254	0.2289	0.0274	–0.3742
		(0.3596)	(0.4016)				
	Static optimisation	0.4402	0.4398	1.0846	1.0846	0.0166	0.0166
		(0.8921)	(0.6114)				
RDCC-GJR	Markowitz	0.8401	–2.4293	1.1557	0.0212	0.0327	–0.3202
		(0.2108)	(0.0150)				
	Static optimisation	0.4066	0.4060	1.0788	1.0787	0.0153	0.0152
		(0.9461)	(0.6324)				

Table 8: Portfolio performance using parameters adopted by GP

This table shows the performance of various portfolio policies using both commodity futures and country ETFs. We use the trading parameters adopted by GP. Specifically, they set $\lambda = 5 \times 10^{-7}$ and $\gamma = 10^{-9}$. The strategies shown include two dyanmic portfolio policies, two static mean-variance portfolio policies with time-varying returns, two norm-constrained mean-variance portfolio policies, and 12 static mean-variance portfolio policies with time-varying covariances. The p-value for the Sharpe ratio difference between each portfolio policy and the GP dynamic portfolio policy is shown in parentheses. For those net RRs that have the same numerical values as the gross RRs, the difference appears from the sixth digit, which is not shown here.

		Gross SR	Net SR	Gross OR	Net OR	Gross RR	Net RR
Dynamic portfolio policies							
GP dynamic optimisation		0.4061	0.3754	1.0845	1.0778	0.0178	0.0164
DMN portfolio policy		0.2867	0.2857	1.0563	1.0561	0.0110	0.0110
		(0.8022)	(0.8601)				
Static portfolio policies with time-varying returns							
Markowitz		0.6315	–13.7415	1.1373	0.0047	0.0293	–0.3757
		(0.0509)	(0.0819)				
Static optimisation		0.1691	0.1675	1.0342	1.0339	0.0072	0.0071
		(0.0549)	(0.0849)				
Short sale-constrained		0.4917	0.4875	1.1150	1.1140	0.0202	0.0201
		(0.7852)	(0.7183)				
Norm-constrained		0.5090	0.5043	1.1153	1.1142	0.0201	0.0199
		(0.7433)	(0.6863)				
Static portfolio policies with time-varying covariances							
RM	Markowitz	0.8940	–2.0349	1.2032	0.0108	0.0411	–0.4317
		(0.2657)	(0.0010)				
	Static optimisation	0.6828	0.6698	1.1420	1.1391	0.0279	0.0273
		(0.5335)	(0.5554)				
RM2006	Markowitz	0.8998	–2.4981	1.1981	0.0110	0.0404	–0.3577
		(0.1798)	(0.0010)				
	Static optimisation	0.6736	0.6642	1.1393	1.1372	0.0274	0.0270
		(0.5554)	(0.5445)				
CCC-GJR	Markowitz	0.9698	–1.5811	1.1959	0.0891	0.0422	–0.4587
		(0.1508)	(0.0010)				
	Static optimisation	0.6528	0.6461	1.1319	1.1305	0.0265	0.0262
		(0.5944)	(0.5614)				
DCC-GJR	Markowitz	0.9186	–1.7211	1.1884	0.1012	0.0403	–0.3938
		(0.1848)	(0.0010)				
	Static optimisation	0.6580	0.6520	1.1346	1.1333	0.0272	0.0269
		(0.5754)	(0.5614)				
RBEKK	Markowitz	0.9476	–0.3317	1.1824	0.3483	0.0385	–0.3341
		(0.1818)	(0.1369)				
	Static optimisation	0.6369	0.6344	1.1270	1.1265	0.0253	0.0252
		(0.5964)	(0.5624)				
RDCC-GJR	Markowitz	0.9218	–1.6917	1.1891	0.1032	0.0402	–0.3937
		(0.2058)	(0.0010)				
	Static optimisation	0.6572	0.6512	1.1344	1.1331	0.0272	0.0270
		(0.5864)	(0.5654)				

Table 9: Portfolio performance with the non-shrinkage covariance matrix

This table shows the performance of various portfolio policies using both commodity futures and country ETFs. The covariance matrix is not shrunk anywhere. The portfolio policies shown include two dynamic portfolio policies, two static mean-variance portfolio policies with time-varying returns, two norm-constrained mean-variance portfolio policies, and 12 static mean-variance portfolio policies with time-varying covariances. The p-value for the Sharpe ratio difference between each portfolio policy and the GP dynamic portfolio policy is shown in parentheses. For those net RRs that have the same numerical values as the gross RRs, the difference appears from the sixth digit, which is not shown here.

		Gross SR	Net SR	Gross OR	Net OR	Gross RR	Net RR
Dynamic portfolio policies							
GP dynamic optimisation		0.5150	0.4836	1.1054	1.0987	0.0231	0.0217
DMN portfolio policy		0.3061	0.3048	1.0599	1.0596	0.0118	0.0117
		(0.6414)	(0.6873)				
Static portfolio policies with time-varying returns							
Markowitz		1.0244	–11.7012	1.2336	0.0015	0.0488	–0.3944
		(0.0779)	(0.1049)				
Static optimisation		0.2390	0.2372	1.0468	1.0464	0.0100	0.0099
		(0.0679)	(0.0969)				
Short sale-constrained		0.4667	0.4629	1.1051	1.1042	0.0185	0.0183
		(0.8841)	(0.9461)				
Norm-constrained		0.5362	0.5319	1.1196	1.1186	0.0211	0.0209
		(0.9610)	(0.8671)				
Static portfolio policies with time-varying covariances							
RM	Markowitz	0.7076	–2.5164	1.1624	0.0014	0.0315	–0.3149
		(0.6134)	(0.0010)				
	Static optimisation	0.6475	0.6310	1.1334	1.1298	0.0262	0.0255
		(0.7602)	(0.7552)				
RM2006	Markowitz	0.6991	–2.7952	1.1527	0.0029	0.0304	–0.2962
		(0.6154)	(0.0030)				
	Static optimisation	0.6305	0.6208	1.1303	1.1281	0.0256	0.0252
		(0.7712)	(0.7642)				
CCC-GJR	Markowitz	0.8742	–1.5890	1.1853	0.0384	0.0394	–0.5264
		(0.3437)	(0.0010)				
	Static optimisation	0.5926	0.5865	1.1197	1.1184	0.0245	0.0242
		(0.8621)	(0.8092)				
DCC-GJR	Markowitz	0.8086	–1.4951	1.1686	0.0441	0.0363	–0.4613
		(0.4186)	(0.0020)				
	Static optimisation	0.6064	0.6010	1.1242	1.1230	0.0241	0.0239
		(0.8252)	(0.7812)				
RBEKK	Markowitz	0.8354	–0.3230	1.1589	0.1057	0.0351	–0.8611
		(0.3946)	(0.3656)				
	Static optimisation	0.6011	0.5987	1.1197	1.1192	1.1197	1.1192
		(0.8312)	(0.8072)				
RDCC-GJR	Markowitz	0.8148	–1.4492	1.1708	0.0444	0.0365	–0.4688
		(0.4056)	(0.0010)				
	Static optimisation	0.6041	0.5987	1.1236	1.1224	0.0241	0.0238
		(0.8501)	(0.8052)				

Table 10: Portfolio performance with the shrinkage covariance matrix by Ledoit and Wolf (2004a)

This table shows the performance of various portfolio policies using both commodity futures and country ETFs. We shrink the covariance matrix of the entire sample towards a constant-correlation matrix according to Ledoit and Wolf (2004a). Each strategy's sample covariance matrix as an estimator is shrunk towards a constant-correlation matrix. The covariance matrix in transaction costs is also shrunk using the approach of Ledoit and Wolf (2004a). The *p*-value for the Sharpe ratio difference between each portfolio policy and the GP dynamic portfolio policy is shown in parentheses. For those net RRs that have the same numerical values as the gross RRs, the difference appears from the sixth digit, which is not shown here.

		Gross SR	Net SR	Gross OR	Net OR	Gross RR	Net RR
Dynamic portfolio policies							
GP dynamic optimisation		0.5154	0.4841	1.1055	1.0988	0.0232	0.0217
DMN portfolio policy		0.2708	0.2697	1.0509	1.0507	0.0104	0.0104
		(0.5495)	(0.6034)				
Static portfolio policies with time-varying returns							
Markowitz		1.0137	–11.8908	1.2307	0.0015	0.0484	–0.3940
		(0.0789)	(0.1059)				
Static optimisation		0.2394	0.2377	1.0468	1.0465	0.0100	0.0100
		(0.0619)	(0.1039)				
Short sale-constrained		0.4672	0.4634	1.1052	1.1043	0.0185	0.0184
		(0.8801)	(0.9441)				
Norm-constrained		0.5367	0.5324	1.1197	1.1186	0.0211	0.0209
		(0.9451)	(0.8731)				
Static portfolio policies with time-varying covariances							
RM	Markowitz	0.7076	–2.5233	1.1624	0.0014	0.0315	–0.3154
		(0.6024)	(0.0010)				
	Static optimisation	0.6475	0.6310	1.1334	1.1297	0.0262	0.0255
		(0.7572)	(0.7702)				
RM2006	Markowitz	0.6991	–2.8010	1.1527	0.0029	0.0304	–0.2958
		(0.6384)	(0.0010)				
	Static optimisation	0.6305	0.6208	1.1303	1.1281	0.0256	0.0252
		(0.8062)	(0.7722)				
CCC-GJR	Markowitz	0.8742	–1.5905	1.1853	0.0382	0.0394	–0.5247
		(0.3057)	(0.0010)				
	Static optimisation	0.5926	0.5865	1.1197	1.1184	0.0245	0.0242
		(0.8631)	(0.8072)				
DCC-GJR	Markowitz	0.8086	–1.4929	1.1686	0.0439	0.0363	–0.4613
		(0.4046)	(0.0010)				
	Static optimisation	0.6064	0.6010	1.1242	1.1230	0.0241	0.0239
		(0.8452)	(0.7822)				
RBEKK	Markowitz	0.8354	–0.3228	1.1589	0.1044	0.0351	–0.8672
		(0.4256)	(0.3576)				
	Static optimisation	0.6011	0.5987	1.1197	1.1192	0.0237	0.0236
		(0.8511)	(0.7962)				
RDCC-GJR	Markowitz	0.8148	–1.4467	1.1708	0.0442	0.0365	–0.4694
		(0.4256)	(0.0010)				
	Static optimisation	0.6041	0.5987	1.1236	1.1224	0.0241	0.0238
		(0.8501)	(0.7882)				

Table 11: Portfolio performance with the shrinkage covariance matrix by Ledoit and Wolf (2004b)

This table shows the performance of various portfolio policies using both commodity futures and country ETFs. We shrink the covariance matrix towards a one-parameter matrix according to Ledoit and Wolf (2004b). Each strategy's sample covariance matrix as an estimator is shrunk towards a one-parameter matrix. The covariance matrix in transaction costs is also shrunk using the approach of Ledoit and Wolf (2004b). The p-value for the Sharpe ratio difference between each portfolio policy and the GP dynamic portfolio policy is shown in parentheses.

		Gross SR	Net SR	Gross OR	Net OR	Gross RR	Net RR
Dynamic portfolio policies							
GP dynamic optimisation		0.5062	0.4750	1.1033	1.0966	0.0227	0.0212
DMN portfolio policy		0.3061	0.3048	1.0599	1.0596	0.0118	0.0117
		(0.6384)	(0.7043)				
Static portfolio policies with time-varying returns							
Markowitz		1.0301	–11.7983	1.2344	0.0015	0.0487	–0.3944
		(0.0659)	(0.1169)				
Static optimisation		0.2333	0.2316	1.0457	1.0454	0.0097	0.0096
		(0.0559)	(0.0989)				
Short sale-constrained		0.4676	0.4638	1.1049	1.1040	0.0185	0.0183
		(0.9181)	(0.9660)				
Norm-constrained		0.5336	0.5293	1.1187	1.1177	0.0208	0.0206
		(0.9351)	(0.8761)				
Static portfolio policies with time-varying covariances							
RM	Markowitz	0.7076	–2.5263	1.1624	0.0014	0.0315	–0.3133
		(0.6094)	(0.0010)				
	Static optimisation	0.6475	0.6310	1.1334	1.1297	0.0262	0.0255
		(0.7522)	(0.7293)				
RM2006	Markowitz	0.6991	–2.8067	1.1527	0.0029	0.0304	–0.2953
		(0.5714)	(0.0150)				
	Static optimisation	0.6305	0.6208	1.1303	1.1281	0.0256	0.0252
		(0.7812)	(0.7233)				
CCC-GJR	Markowitz	0.8742	–1.5899	1.1853	0.0382	0.0394	–0.5253
		(0.3217)	(0.0010)				
	Static optimisation	0.5926	0.5865	1.1197	1.1184	0.0245	0.0242
		(0.8432)	(0.7652)				
DCC-GJR	Markowitz	0.8086	–1.4942	1.1686	0.0438	0.0363	–0.4621
		(0.4196)	(0.0010)				
	Static optimisation	0.6064	0.6010	1.1242	1.1230	0.0241	0.0239
		(0.8172)	(0.7952)				
RBEKK	Markowitz	0.8354	–0.3234	1.1589	0.1044	0.0351	-0.8671
		(0.3786)	(0.4006)				
	Static optimisation	0.6011	0.5987	1.1197	1.1192	0.0237	0.0236
		(0.8452)	(0.7912)				
RDCC-GJR	Markowitz	0.8148	–1.4480	1.1708	0.0441	0.0365	–0.4688
		(0.3746)	(0.0010)				
	Static optimisation	0.6041	0.5987	1.1236	1.1224	0.0241	0.0238
		(0.8192)	(0.7732)				

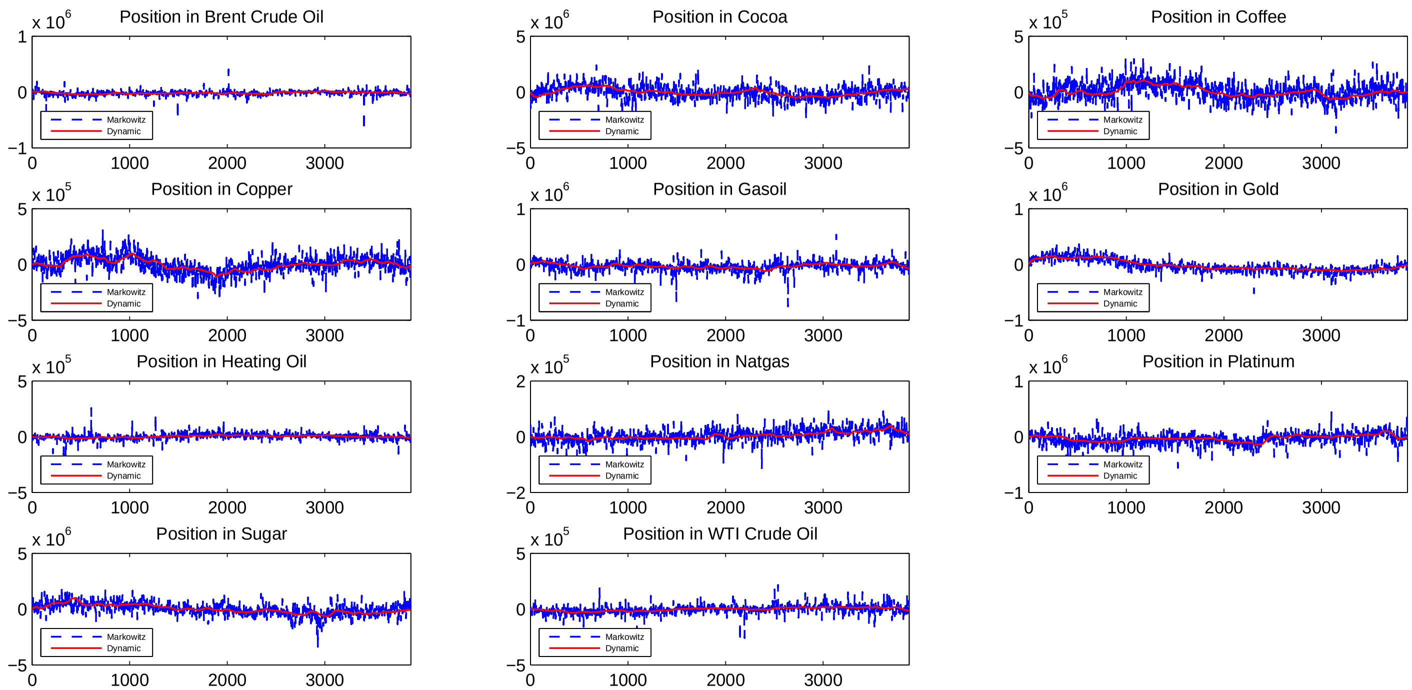

Figure 1: Position of each commodity futures in the Markowitz and GP dynamic portfolios

This figure shows the individual asset position across the full sample period of 11 commodity futures in the Markowitz and GP dynamic portfolios with time-varying returns and time-invariant covariances. The period starts on 01/09/1998 and ends on 02/09/2014. There are 3881 observations involved.

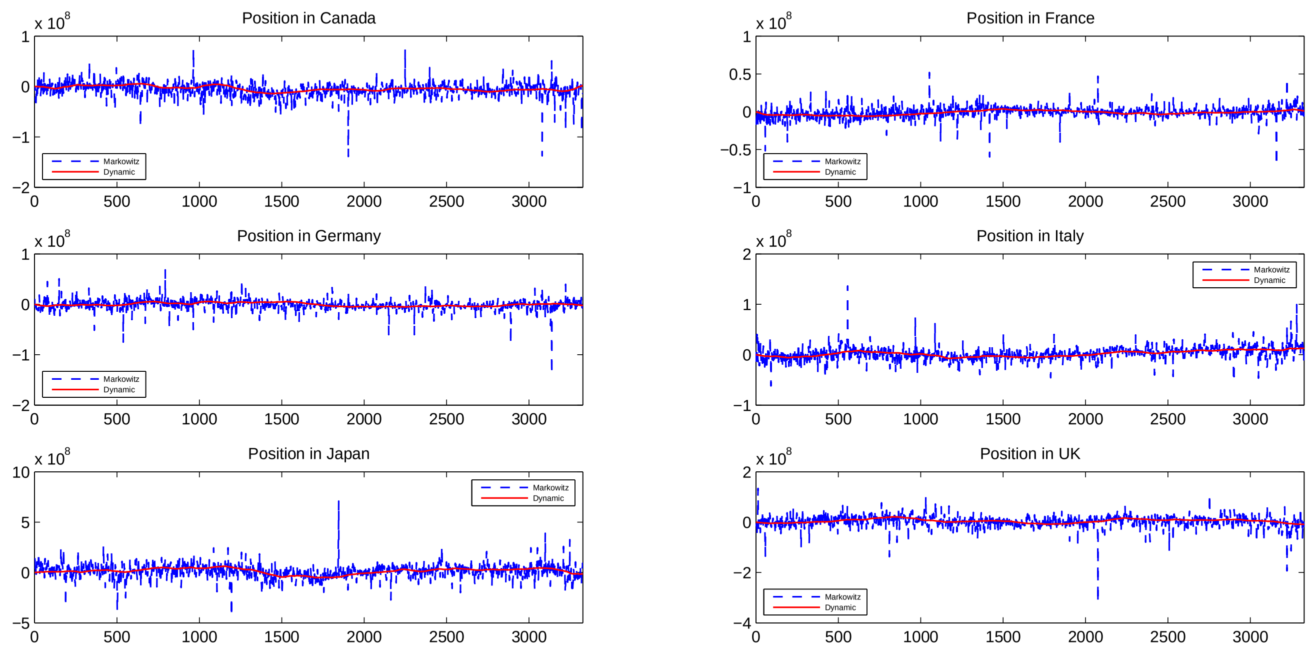

Figure 2: Position of each ETF in the Markowitz and GP dynamic portfolios

This figure shows the individual asset position across the full sample period of six country ETFs in the Markowitz and GP dynamic portfolios with time-varying returns and time-invariant covariances. The period starts on 14/05/2001 and ends on 02/09/2014. There are 3326 observations involved.

References

Asness, C. S., T. J. Moskowitz, and L. H. Pedersen (2013). Value and momentum everywhere. *Journal of Finance 68*(3), 929–985.

Bollerslev, T. (1990). Modelling the coherence in short-run nominal exchange rates: A multivariate generalized ARCH model. *Review of Economics and Statistics 72*(3), 498–505.

Breen, W. J., L. S. Hodrick, and R. A. Korajczyk (2002). Predicting equity liquidity. *Management Science 48*(4), 470–483.

Collin-Dufresne, P., K. Daniel, C. C. Moallemi, and M. Saglam (2015). Strategic asset allocation with predictable returns and transaction costs. *Working Paper, Columbia University*.

Davis, M. H. A. and A. R. Norman (1990). Portfolio selection with transaction costs. *Mathematics of Operations Research 15*(4), 676–713.

DeMiguel, V., A. Martin-Utrera, and F. J. Nogales (2015). Parameter uncertainty in multiperiod portfolio optimization with transaction costs. *Journal of Financial and Quantitative Analysis 50*(6), 1443–1471.

DeMiguel, V., X. Mei, and F. J. Nogales (2016). Multiperiod portfolio optimization with multiple risky assets and general transaction costs. *Journal of Banking and Finance 69*, 108–120.

Engle, R. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business and Economic Statistics 20*(3), 339–350.

Engle, R. and R. Ferstenberg (2007). Execution risk. *Journal of Portfolio Management 33*(2), 34–44.

Engle, R., R. Ferstenberg, and J. Russell (2012). Measuring and modeling execution cost and risk. *Journal of Portfolio Management 38*(2), 14–28.

Engle, R. and K. Kroner (1995). Multivariate simultaneous generalized ARCH. *Econometric Theory 11*(1), 122–150.

Erb, C. B. and C. R. Harvey (2006). The strategic and tactical value of commodity futures. *Financial Analysts Journal 62*(2), 69–97.

Fabozzi, F. J., S. M. Focardi, and P. N. Kolm (2010). *Quantitative equity investing: Techniques and strategies*. Hoboken: Wiley.

Fleming, J., C. Kirby, and B. Ostdiek (2001). The economic value of volatility timing. *Journal of Finance 56*(1), 329–352.

Frost, P. A. and J. E. Savarino (1988). For better performance: Constrain portfolio weights. *Journal of Portfolio Management 15*(1), 29–34.

Garleanu, N. and L. H. Pedersen (2013). Dynamic trading with predictable returns and transaction costs. *Journal of Finance 68*(6), 2309–2340.

Glosten, L., R. Jagannathan, and D. E. Runkle (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *Journal of Finance 48*(5), 1779–1801.

Greenwood, R. (2005). Short- and long-term demand curves for stocks: Theory and evidence on the dynamics of arbitrage. *Journal of Financial Economics 75*(3), 607–649.

Hong, L. (2004). Optimal consumption and investment with transaction costs and multiple risky assets. *Journal of Finance 59*(1), 289–338.

Hong, L. and M. Loewenstein (2002). Optimal portfolio selection with transaction costs and finite horizons. *Review of Financial Studies 15*(3), 805–835.

Jagannathan, R. and T. Ma (2003). Risk reduction in large portfolios: Why imposing the wrong constraints helps. *Journal of Finance 58*(4), 1651–1684.

Keating, C. and W. F. Shadwick (2002). A universal performance measure. *Journal of Performance Measurement 6*(3), 59–84.

Ledoit, O. and M. Wolf (2004a). Honey, I shrunk the sample covariance matrix. *Journal of Portfolio Management 30*(4), 110–119.

Ledoit, O. and M. Wolf (2004b). A well-conditioned estimator for large-dimensional covariance matrices. *Journal of Multivariate Analysis 88*(2), 365–411.

Ledoit, O. and M. Wolf (2008). Robust performance hypothesis testing with the Sharpe ratio. *Journal of Empirical Finance 15*, 850–859.

Markowitz, H. M. (1952). Portfolio selection. *Journal of Finance 7*(1), 77–91.

Merton, R. C. (1980). On estimating the expected return on the market an exploratory investigation. *Journal of Financial Economics 8*(4), 323–361.

Moallemi, C. C. and M. Saglam (2015). Dynamic portfolio choice with linear rebalancing rules. *Working Paper, Columbia University.*

Noureldin, D., N. Shephard, and K. Sheppard (2014). Multivariate rotated ARCH models. *Journal of Econometrics 179*(1), 16–30.